Sur un test temps-fréquence de stationnarité

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On a time-frequency test of stationarity

Extended English summary

1 Introduction

Whereas the usual concept of stationarity is a stochastic one that is well-defined (as the strict invariance of statistical properties with respect to some absolute time), its practical use is somewhat different, taking often into account—though implicitly—observation scales or frequency bands, as well as accommodating for periodicities in deterministic signals. As for the notion of dimension that may vary depending on the observation scale [3], it thus turns out that the very same signal can be considered as stationary or not, depending on the way we look at it. A typical example is given by speech that is usually considered as nonstationary (resp. stationary) at the scale of some seconds (resp. tens of milliseconds), while turning again to nonstationary at the scale of a few milliseconds within voiced segments. Recognizing this situation calls therefore for an operational framework aimed at making of stationarity a *relative* concept by incorporating the observation scale in its definition, while encompassing in a common perspective stochastic and deterministic descriptions. Revisiting this way the concept of stationarity, the purpose of this paper is to go further by designing a meaningful test for its assessment.

2 General framework

The rationale of the proposed approach is based on the fact that, in both stochastic and deterministic contexts, a signature of "stationarity" is that a well-defined time-varying spectrum just reduces to the the ordinary spectrum at all time instants [1].

2.1 Time-frequency

The first ingredient is therefore a suitable time-varying spectrum, here chosen as the Wigner-Ville spectrum estimated by means of multitaper spectrograms, see eq.(1), for a sake of variance reduction without some extra time averaging that would possibly smooth out nonstationarities [6,7].

2.2 Surrogates

Given an observation scale, the basic idea is to characterize stationarity as the identity between *local* features (frequency slices) of the time-varying spectrum and *global* ones obtained by marginalization [2,4,5]. In practice, the difference will never be zero and, to give it a statistical significance, it is proposed to create *from the data itself* a stationarized reference. This is achieved by the technique of surrogate data [8,9,10] that essentially amounts to scramble the spectrum phase (in which possible nonstationarities are coded) of the signal under test while keeping its magnitude unchanged, see Figure 1.

2.3 Distances

Differences between local and global spectral properties are quantified by means of a dissimilarity measure [11], see eq.(2). A companion study has evidenced that a good choice is to mix a Kullback-Leibler divergence with a log-spectral deviation [12].

3 Stationarity test

3.1 Principle

The stationarity test itself is described in eqs.(3)-(6), with the threshold γ derived from the empirical distribution attached to the surrogates features.

3.2 Null hypothesis of stationarity

An experimental study shows that the above mentioned distribution can be fairly well approximated by a Gamma distribution (see Figure 2), allowing for a parametric approach. As evidenced in Figures 3 and 4 for various window lengths, the maximum likelihood estimation of the two Gamma parameters converge quickly to an asymptotic value when the number of surrogates J is increased. As a rule-of-thumb, the value $J \sim 50$ can be retained for a good trade-off between complexity and accuracy. Recalling that surrogates are supposed to define the null hypothesis of stationarity, it is worth checking that, in a stationary situation, the test behaves as expected from the viewpoint of the imposed false alarm rate fixed by the threshold. This is illustrated in Figure 5 (Monte-Carlo simulation with an AR(2) model), with a result that is slightly pessimistic, the observed false alarm being of about 6.5% for a confidence level fixed to 5%.

3.3 Index and scale of nonstationarity

Beyond its binary nature, the test defined in eq.(6) also allows for a quantified measure of a *degree of nonstationarity* according to eq.(7). Moreover, since the overall procedure is dependent on the spectrogram window length, it is also possible to conduct the test for a family of such window lengths, ending up with a typical *scale of nonstationarity* as in eq.(8).

4 Example

Figure 6 provides an example aimed at supporting the effectiveness of the proposed approach. The analyzed signal consists in the superimposition of two components, one of which being a tone that can be considered as stationary whatever the observation scale is whereas the second one is a sinus FM whose (non) stationary character depends on the observation scale. Running the test in the two corresponding subbands as a function of both the observation length N_x and the relative analysis scale N_h/N_x ends up with results in clear agreement with physical interpretation.

5 Conclusion

A new approach has been proposed for testing stationarity relatively to an observation scale. The approach operates in the time-frequency plane by comparing local and global features, the null hypothesis of stationarity being characterized by surrogate data directly derived from the data under test. The method reported here is basically based on distances but it allows for a number of variations, such as the recourse to machine learning methods (one-class SVM [13]) by considering the family of surrogates as a learning set.