Directed Percolation arising in Stochastic Cellular Automata

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Abstract. Cellular automata are both seen as a model of computation and as tools to model real life systems. Historically they were studied under synchronous dynamics where all the cells of the system are updated at each time step. Meanwhile the question of probabilistic dynamics emerges: on the one hand, to develop cellular automata which are capable of reliable computation even when some random errors occur [24,14,13]; on the other hand, because synchronous dynamics is not a reasonable assumption to simulate real life systems.

Among cellular automata a specific class was largely studied in synchronous dynamics : the elementary cellular automata (ECA). These are the "simplest" cellular automata. Nevertheless they exhibit complex behaviors and even Turing universality. Several studies [20,7,8,5] have focused on this class under α-asynchronous dynamics where each cell has a probability α to be updated independently. It has been shown that some of these cellular automata exhibit interesting behavior such as phase transition when the asynchronicity rate α varies.

Due to their richness of behavior, probabilistic cellular automata are also very hard to study. Almost nothing is known of their behavior [20]. Understanding these "simple" rules is a key step to analyze more complex systems. We present here a coupling between oriented percolation and ECA 178 and confirms observations made in [5] that percolation may arise in cellular automata. As a consequence this coupling shows that there is a positive probability that the ECA 178 does not reach a stable configuration with positive probability as soon as the initial configuration is not a stable configuration and $\alpha > 0.996$. Experimentally, this result seems to stay true as soon as $\alpha > \alpha_c \approx 0.5$.

1 Introduction

A cellular automaton is a process where several cells, characterized by a state, evolve according to the states of their neighboring cells. Cellular automata can both model parallel computing and real life systems [22] .

Historically they have been studied under synchronous dynamics where all the cells update at the same time. Meanwhile models of probabilistic cellular

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automata have emerged. Several studies focus on a model of cellular automata evolving synchronously but where some random errors can occur. In [24], Toom gives a 2D cellular automata capable of remembering one bit of information in presence of random error. This result is used in [14] to develop a 3D cellular automaton capable of reliable computation. Later on Gács proves the existence of a 1D cellular automaton exhibiting the same reliability [13]. In [1] the authors try to apply the mean field approach on a probabilistic model of cellular automata and show that complex behaviors cannot be explained by this method.

Several empirical studies have shown that the behavior of cellular automata changes drastically under asynchronous dynamics [2,3,6,17,23]. Only few theoretical results are known. Mainly, either they concern specific cellular automata or show that it is difficult to describe the global behavior of cellular automata under probabilistic updates [1,12,13,10,11,20,7,8,21].

Percolation theory was introduced to model the fact that a liquid or a gas can flow through a solid due to porosity. Other applications were found for this model and it has been extensively study in the last decades on which probability theory made a lot of progress. A good introduction to percolation theory can be found in [15]. Ising models, Potts models and percolation were unified into the random cluster model [9,16]. We define here a coupling between oriented percolation and a "simple" probabilistic cellular automaton. This coupling shows that "simple" rules may embed very complex phenomena. The link between elementary cellular automata and percolation theory was already observed in experiments in [5] but it is the first time that a correlation is proved.

The probabilistic dynamics we study here is the α-*asynchronous dynamics* where each cell has an independent probability α to be updated and a probability $1 - \alpha$ to stay in its current state at each time step. A particular class of cellular automata is the 256 elementary cellular automata (ECA). This class gathers the "simplest" cellular automata : cells are placed on a line, they are characterized by a state which is 0 (white) or 1 (black) and they can communicate only with their two closest neighbors. Nevertheless, studies have shown that this class exhibits a wide range of behavior including Turing universality [4]. Even if this class seems simple, questions remain open in the deterministic synchronous case such as the intrinsic universality of ECA 110 [18]. Current works study this class under α-asynchronous dynamics [20,7,8], in particular the 64 cellular automata for which the two configurations all black and all white are stable. The understanding of these "simple" rules is a key step to understand more complex phenomena. First in [7] it was shown that when only one cell is updated at each time step (which may be under certain circumstances the limit as $\alpha \to 0$) the behavior of these cellular automata are similar to behavior of coupon collectors or random walks. In [8], the behavior of some of these cellular automata has been determined under α-asynchronous dynamics, and the authors have isolated some cellular automata exhibiting rich behavior such as phase transitions. The elementary cellular automaton studied here ECA 178, FLIP-IF-NOT-ALL-EQUAL is one of these complex automata. These automata are hard to study and only few results are known yet. Here we present a new

result on the ECA 178 and show that there exists a positive probability that the process will never reach a stable configuration when $\alpha > 0.996$ even when the initial configuration contains only one black cell. This is the first result proved for this automaton. Moreover ongoing work shows that ECA 178 arises naturally in the study of 2D Minority [21] which is a model of anti-ferromagnetism in physics [19] and also one of the simplest non-monotonic gene network model in biology. Since the proof of this result is based on a coupling between the space-time diagram of ECA 178 and oriented percolation on a graph, it tends to support the fact that the behavior of this automaton is indeed complex since very little is known on directed percolation yet.

2 Asynchronous cellular automata

2.1 Definition

We give here a formal definition of ECA 178. The next part presents informally its behavior and the underlying difficulties of its analysis.

Definition 1 (Configuration). *We denote by $\mathbb{Z}$ the set of* cells *and $Q = \{0, 1\}$ the set of* states *(0 stands for white and 1 for black in the figures). The* neighborhood *of a cell i consists of the cells $i - 1$, i and $i + 1$. A* configuration c *is a function $c : \mathbb{Z} \to Q$; c_i is the state of the cell i in configuration c.*

Definition 2 (ECA 178: FLIP-IF-NOT-ALL-EQUAL). *The* rule *of a cellular automaton is a function which associates a state to a neighborhood. The rule δ of the ECA 178 is defined as follows:*

$$\delta(c_{i-1}, c_i, c_{i+1}) = \begin{cases} c_i & \text{if } c_{i-1} = c_i = c_{i+1} \\ 1 - c_i & \text{otherwise} \end{cases}$$

Time is discrete and in the classic deterministic synchronous dynamics all the cells of a configuration are updated at each time step according to the transition rule of the cellular automaton (see figure 1). Here we consider a stochastic asynchronous dynamics where only a random subset of cells is updated at each time step.

Definition 3 (Asynchronous dynamics). *Given $0 < \alpha \leqslant 1$, we call α-asynchronous dynamics the following process : time is discrete and c^t denotes the random variable for the configuration at time t. The configuration c^0 is the initial configuration. The configuration at time $t + 1$ is the random variable defined by the following process : each cell has independently a probability α to be updated according to the rule δ (we say that the cell fires at time t) and a probability $1 - \alpha$ to remain in its current state. A cell is said active if its state changes when fired.*

Figure 1 presents different space-time diagrams of ECA 178 for different values of α. The initial configuration consists in one single black cell and is displayed horizontally at the bottom of the diagram (time flows upwards).

$\alpha = 0$ $\alpha = 0.1$ $\alpha = 0.25$ $\alpha = 0.5$ $\alpha = 0.75$ $\alpha = 0.9$ synchronous dynamics
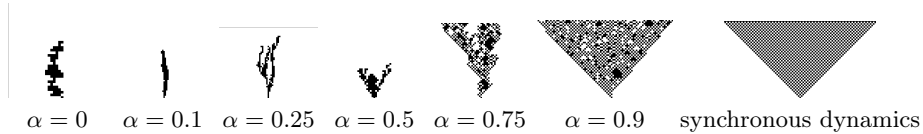
Fig. 1. ECA 178 under different dynamics ($\alpha = 0$ stands for fully asynchronous dynamics, for this diagram only one every 50 time step is displayed).

Definition 4 (Stable configuration). *A configuration c is a stable if for all $i \in \mathbb{Z}$, $\delta(c_{i-1}, c_i, c_{i+1}) = c_i$.*

ECA 178 (FLIP-IF-NOT-ALL-EQUAL) admits only two stable configurations : the configurations all white and all black.

Definition 5 (Convergence). *We say that a random sequence $(c^t)_{t \geqslant 0}$ defined by ECA 178 converges under α-asynchronous dynamics if there exists $t < \infty$ such that c^t is a stable configuration. We denote by $P_\alpha(c^0)$ the probability that such a t exists.*

From the definition of stable configuration, it follows that if there exists t such that c^t is a stable configuration then for all $t' \geqslant t$ the configuration $c^{t'}$ is the same stable configuration. Note that since the configuration is infinite, only specific initial configurations may converge with positive probability. Here we will consider only a particular initial configuration which is very "close" to a stable configuration and show that when α is large enough, being this close is not enough to guarantee convergence almost surely.

Definition 6 (Initial configuration). *We define c^{init} as the configuration where $c_0^{init} = 1$ and for all $i \neq 0$, $c_i^{init} = 0$.*

From now on, we will consider that the initial configuration is always c^{init}. The configuration c^{init} differs from the configuration all white by only one cell. Nevertheless we show the following result :

Theorem 1 (Main result). *If $\alpha \geqslant \sqrt[3]{80/81} \approx 0.996$ then $P_\alpha(c^{init}) < 1$.*

Section 4 is dedicated to the proof of this result. This is the first result on ECA 178. This result shows that this rule can exhibit very complex behavior and shows how simple rules may turn out to be hard to analyze. Before the proof, the following section exposes experimental results on the behavior of ECA 178.

2.2 Discussion

In [8] it was conjectured that ECA 178 admits a phase transition which occurs experimentally at $\alpha = \alpha_c \approx 0.5$. Figure 1 illustrates the changes in the space time diagrams of ECA 178 when α varies. In [7] it was proven that this automaton behaves as a non-biased random walk on a finite configuration with

periodic boundary condition under the fully asynchronous dynamics (when only one random cell fires at each time step). This proof can easily be extended to infinite configurations when $c^0 = c^{init}$ to prove that its converges in polynomial time to all white almost surely under the fully asynchronous dynamics.

When $0 < \alpha < \alpha_c$, despite of the fact that some small "errors" may occur, the global behavior seems to be similar to the fully asynchronous dynamics. When $\alpha > \alpha_c$ the behavior changes drastically : an alternating background pattern (0101010) appears and extends quickly in expense of the black and white regions preventing the configuration from ever reaching a fixed point. Cells inside a big white or black region are inactive whereas cells of a 010101 region are all active. When α is very small, regions of 010101 are highly unstable and the presence of patterns 010101 is marginal. Nevertheless this pattern does not exist in the fully asynchronous case and since the study of this dynamics relies on a perfect symmetry between black and white regions, the emergence of a third kind of region, which behave drastically differently, prevents us from deriving a lower bound from the fully asynchronous dynamics. We believe that any lower bound on α_c would be a huge achievement.

We prove next that when $\alpha > 0.996$ there is a strictly positive probability that the process will not reach the stable configuration all white even if only one cell is black in the initial configuration. The global behavior here is thus no more related to a unbiased random walk. Since rule 178 acts symmetrically on black and white states the same result holds when the initial configuration has only one white cells. Big white regions and big black regions tends to disappear in favor of 0101 patterns and our result confirms the fact that the pattern 0101 ends up dominating in the configuration when $\alpha > 0.996$. Finally our proof is based on a coupling between cellular automaton and oriented percolation. This correlation was already spotted in [5] but it is the first time that it is theoretically proved and the emergence of oriented percolation in ECA 178 is also a strong evidence of the richness of its behavior.

3 Oriented percolation on $(\mathbb{Z}^+)^2$

Definition 7 (Oriented bond percolation). *Consider a probability p and the randomly labeled graph $\mathbb{L}(p) = ((\mathbb{Z}^+)^2, \mathbb{E})$ where $(\mathbb{Z}^+)^2$ is called the set of* sites *and $\mathbb{E}$ the set of* bonds. *For all $i, j \in \mathbb{Z}^+$, there are oriented bonds between site (i, j) and sites $(i+1, j)$ and $(i, j+1)$. Each bond has independently a probability p to be labeled* open *and a probability $1 - p$ to be labeled* closed.

Figure 2 illustrates several examples of oriented percolation for different values of p. Only open bonds are shown in the figure. The main question of percolation theory is the size of the open cluster.

Definition 8 (Open cluster). *We denote by C the* open cluster *of site $(0,0)$: a site (i, j) is in C if and only if there is an oriented path from site $(0,0)$ to site (i, j) only made of open bonds. We call $\theta(p)$ the propability that C is infinite (*i.e. *that there exists an infinite open path from cell $(0,0)$).*

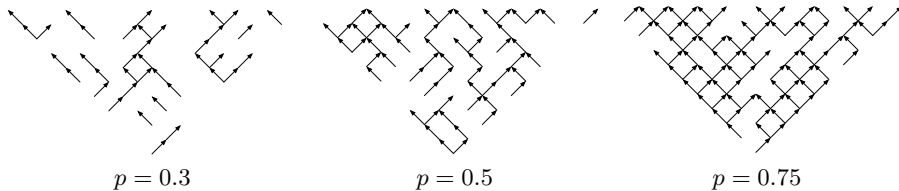$p = 0.3$ $p = 0.5$ $p = 0.75$

Fig. 2. Oriented percolation with different probability of open bonds.

It is easy to show that $\theta(p)$ is an increasing function of p. Moreover there exists a critical value p_c such that $\theta(p) > 0$ for $p > p_c$. Bounds on this critical value can be found in [15].

Theorem 2 (Critical value [15]). *There exists a critical value p_c such that if $p < p_c$ then $\theta(p) = 0$, and if $p > p_c$ then $\theta(p) > 0$. Moreover, $0.6298 < p_c < 2/3$.*

4   Coupling cellular automaton 178 with percolation

There is no notion of time in percolation but the height will stand for it in our coupling. Indeed in order to know if a site of height $t+1$ is in the open cluster or not, we only need information about sites of height t. In a cellular automata in order to know the state of a cell at time $t + 1$, we only need to know the states of cells at time t.

Definition 9 (Height). *The* height *of a percolation site (i, j) is the length of any path from $(0,0)$ to (i, j) in $\mathbb{L}(p)$, that is to say $i + j$. We denote by C^t the sites of height t which are in the open cluster C. The height of a bond is the height of its origin.*

Definition 10 (Candidate). *A site is a* candidate *of height $t+1$ if and only if at least one of its predecessors is in C^t. We denote by $\hat{C}^{t+1}$ the set of candidates of height of $t + 1$.*

Clearly for all $t > 0$ we have $C^t \subset \hat{C}^t$. Figure 4 illustrates the notion of height and candidate.
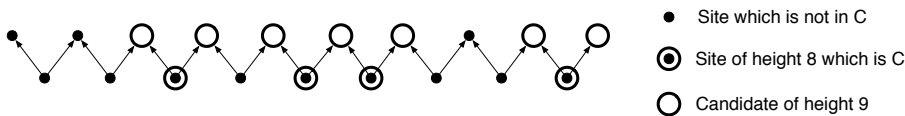

●   Site which is not in C

◉   Site of height 8 which is C

○   Candidate of height 9

Fig. 3. An example of sites and candidates of height 8 and 9. All bonds (indifferently open or closed) are represented.

In this part, $(c^t)_{t \geqslant 0}$ denotes the random sequence of configurations updated according to rule 178 under α-asynchronous dynamics where $c^0 = c^{init}$.

Definition 11 (Mapping). *We define the set of cells $\mathbb{T}^t$ of configuration c^t as follows : cell c_i^t is in $\mathbb{T}^t$ if and only if $-t \leqslant i \leqslant t$ and $i \equiv t \bmod 2$. Let $\mathbb{T} = \cup_{t=0}^{\infty} \mathbb{T}^t$. We define $g : (\mathbb{Z}^+)^2 \to \mathbb{T}$ as the bijection which associates the percolation site (i, j) to the cell $2i - \lceil \frac{i+j}{2} \rceil$ of configuration c^{i+j}. From these definitions, it follows that the image of the sites of height t in $\mathbb{L}(p)$ by g are the cells $\mathbb{T}^t$ in configuration c^t.*

Figure 4 shows the mapping of $\mathbb{L}(p)$ on a space-time diagram. We can notice that the cells of $\mathbb{T}$ correspond exactly to the black cells of the space-time diagram of figure 1 under synchronous dynamics. Our aim is to define a coupling such that the corresponding cells of sites in C will always be active. The following criterion formulates this property.
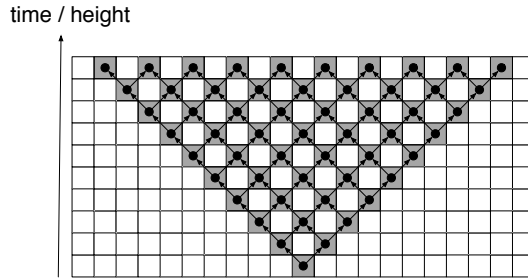
time / height

Fig. 4. Mapping of the graph $\mathbb{L}(p)$ on the space-time of a cellular automaton; the cells of $\mathbb{T}$ are colored in black.

Definition 12 (Correspondence criterion). *We say that a space-time diagram $(c^t)_{t>0}$ and a labelled directed graph $\mathbb{L}(p)$ satisfy the* correspondence criterion *at time t if and only if the cells of $g(C^t)$ are all active. We say that they satisfy the correspondence criterion if and only if they satisfy the correspondence criterion for all $t \geqslant 0$ (i.e. the cells of $g(C)$ are all active).*

Since in c_0^{init} site $(0, 0) \in C$ is active, the configuration $c^0 = c^{init}$ and any randomly labeled directed graph $\mathbb{L}(p)$ always satisfy the correspondence criterion at time 0. Moreover if they satisfy the correspondence criterion and the open cluster is infinite in $\mathbb{L}(p)$ then for all t, c^t admits at least one active cell and thus the sequence (c^t) does not converge. Figure 5 gives a configuration which satisfies the correspondence criterion with the directed graph of figure 4 at time 8.

Consider a configuration c^t such that the correspondence criterion is true at time t. We want in our coupling that the correspondence criterion stays true at time $t+1$. Consider a site in $\tilde{C}^{t+1}$, this site may be in C. If it is the case, the cell c_i^{t+1} corresponding to this site has to be active. Thus, we have to focus on

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cell c_i^t at time t and design a coupling such that this cell is active at time $t + 1$ if the corresponding site is in the open cluster. We say that there is a constraint on this cell.
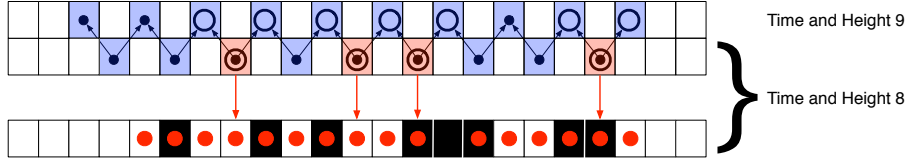

Fig. 5. Cells of $\mathbb{T}^8$ and $\mathbb{T}^9$ are colored in red and blue. Active cells are marked by a red dot. Red arrows link the sites of C^8 to their corresponding cells in c^8. In this example, these cells are active in c^8. Thus the correspondence criterion is verified at time 8.

Definition 13 (Constrained cells). *A cell c_i^t is* constrained *at time t if and only if $c_i^{t+1} \in \mathbb{T}$ and $g^{-1}(c_i^{t+1})$ is in $\hat{C}^{t+1}$.*

We may have to force a constrained cell to be active at time $t + 1$ in our coupling. This turn out to be possible because a constrained cell always has an active cell in its neighborhood when the correspondence criterion is verified (Lemma 1). It follows that one can associate to each constrained cell a partner which is an active cell in its neighborhood (Definition 14). Firing these two cells makes the constrained cell active at time $t + 1$ (Lemma 2) which can be coupled with the opening of the bonds when α is large enough (Theorem 3).

Lemma 1. *If $(c^t)_{t \geqslant 0}$ and $\mathbb{L}(p)$ verify the correspondence criterion at time t' each constrained cell of configuration $c^{t'}$ has an active neighbor.*

Proof. Suppose that cell c_i^t is constrained at time t. Then site $(k, l) = g^{-1}(c_i^{t+1})$ is in $\hat{C}^{t+1}$. Thus at least one of the two sites $(k, l - 1)$ and $(k - 1, l)$ is in C^t. Since the correspondence criterion is verified at time t, at least one of the two cells $g(k, l - 1) = c_{i+1}^t$ and $g(k - 1, l) = c_{i-1}^t$ is active.$\square$

Definition 14 (Partner). *The* partner *of a constrained cell c_i^t is defined as follows :*

- if c_i^t is the pointed cell in the neighborhood ▆▆▢▢ *or* ▢▆▆▆ *then its partner is c_{i-1}^t.*

- if c_i^t is the pointed cell in the neighborhood ▆▆▢ *or* ▆▆▆▆▢ *then its partner is c_{i+1}^t.*

Figure 6 illustrates the constrained cells of Figure 4 and their partners. Since two cells of $\mathbb{T}$ cannot be neighbors, two constrained cells cannot be neighbors. Moreover since the partner of a constrained cell is a neighbor of this cell then a cell cannot be at the same time constrained and the partner of an other cell. Note that however a cell may be the partner of two constrained cells.
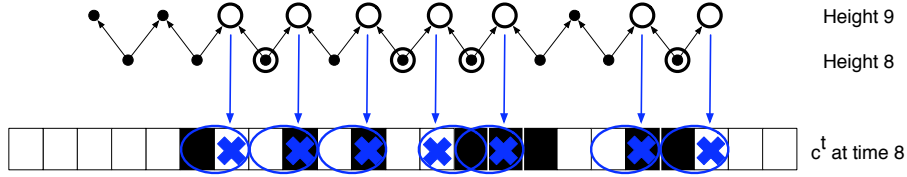
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Fig. 6. Blue arrows map the site of $\hat{C}^9$ to their corresponding constrained cells. These constrained cells are marked by a blue cross. Rounds surround a constrained cell and its partner.

Lemma 2. *For all $t' > 0$, if $(c^t)_{t \geqslant 0}$ and $\mathbb{L}(p)$ verify the correspondence criterion at time t', each constrained cell at time t' has a partner and if these two cells fire at time t' then the constrained cell is active at time $t' + 1$.*

Proof. Suppose that cell c_i^t is constrained at time t. Then w.l.o.g. we suppose that the state of c_i^t is 1 (the other case is symmetric). We do a case study on the states of its neighboring cells and show that in every cases, if c_i^t and its partner fire then c_i^{t+1} is active. This case study is done in figure 7. $\square$
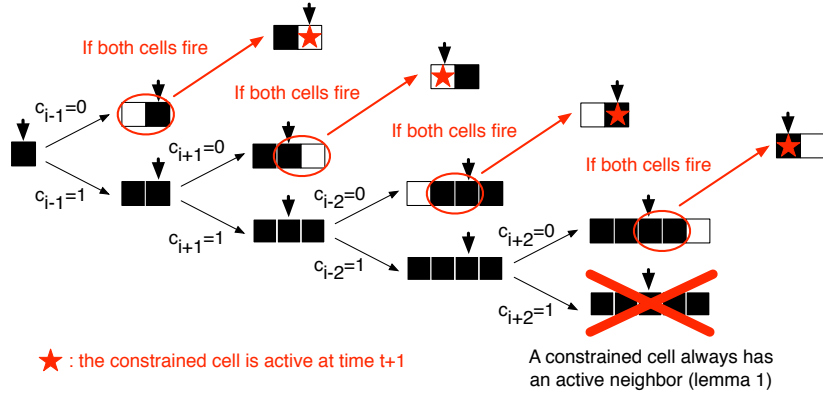

Fig. 7. proof of lemma 2

Theorem 3. *If $\alpha \geqslant \sqrt[3]{1 - (1 - p_c)^4}$ then $P_\alpha(c^{init}) < 1$.*

Proof. Consider a random sequence $(c^t)_{t \geqslant 0}$ of configurations updated according to rule 178 under α-asynchronous dynamics where $c^0 = c^{init}$ and a randomly labeled graph $\mathbb{L}(p)$. Assume $\alpha \geqslant \sqrt[3]{1 - (1 - p)^4}$. We design a coupling between these two processes.

The coupling is defined recursively for all t between c^t and bonds of height t in $\mathbb{L}(p)$. At time t we have to label the edges of height t of $\mathbb{L}$ such that each edge
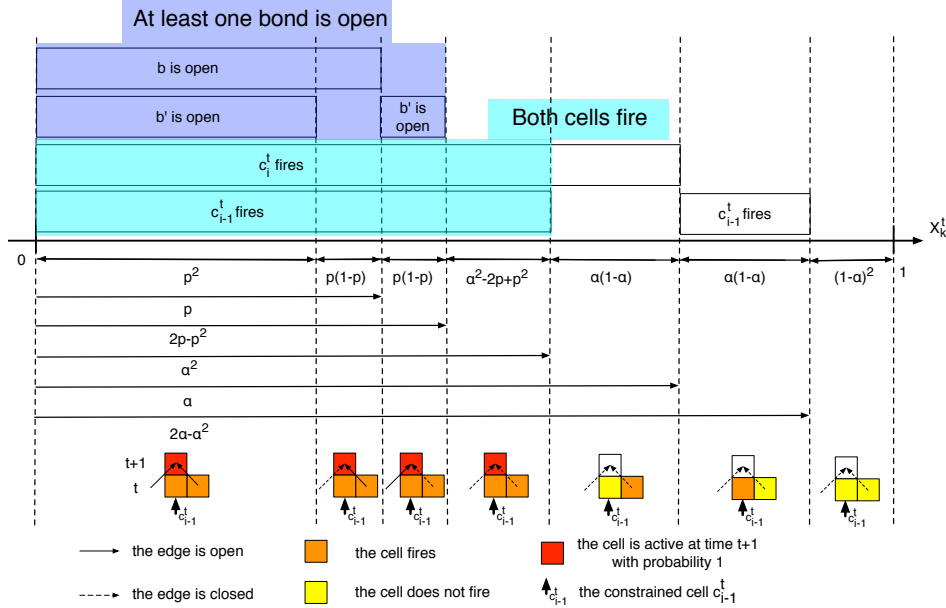
is open with independent probability p. We have to chose which cells fire such that each cell fires with independent probability α. We also design our coupling such that if at least one of the two bonds which ends to a site of $\hat{C}^{t+1}$ is open then the corresponding cell c_i^{t+1} is active. In order to do so, we use Lemma 2 and assuming that the correspondence criterion is true at time t, we force the constrained cell c_i^t and its partner to fire. Since the correspondence criterion is true at time 0, it will recursively stay true for all t.

To achieve this, we partition the cells of c^t and the bonds of height t into buckets. Each bucket consists in :

- type 1 : a unique cell c_i^t if it is neither constrained nor the partner of any other cell.
- type 2 : a unique bond b if it does not end to a site of $\hat{C}^{t+1}$.
- type 3 : two cells c_i^t and $c_{i+\epsilon}^t$ with $\epsilon = 1$ or -1 and the two bonds of height t pointing to the site corresponding to cell $c_{i+\epsilon}^{t+1}$ if c_i^t is the partner of only one constrained cell $c_{i+\epsilon}^t$.
- type 4 : three cells c_{i-1}^t, c_i^t and c_{i+1}^t and the four bonds of height t pointing to the sites corresponding to cells c_{i-1}^{t+1} and c_{i+1}^{t+1} if c_i^t is the partner of the two constrained cells c_{i-1}^t and c_{i+1}^t.

One can easily verify that each bond of height t and each cell of c^t belongs to exactly one bucket. We assign to each bucket k an independent random variable X_k^t uniformly distributed in $[0, 1]$. Now we define the coupling within each bucket:

- type 1 : if $X_k^t < \alpha$ then c_i^t fires and if $X_k^t > \alpha$ then c_i^t does not fire.
- type 2 : if $X_k^t < p$ then b is open and if $X_k^t > p$ then b does is closed.
- type 3 : bounds are open and cells fires according to this diagram :

Each bond is open with probability p. Bonds b and b' are opened with probability p^2, b is open and b' is closed with probability $p(1-p)$ and b is closed and b' is open with probability $p(1-p)$. Thus each bond is open independently of the other. Same thing holds for the cells. Since $\alpha \geqslant \sqrt[3]{1-(1-p)^4}$, we have $\alpha^2 - 2p + p^2 > 0$ and the probability that at least one bond is open is less than the probability that both cells fire.

- type 4 : As in the previous case we define a coupling which respect the distribution of probability and the independence of updates/labeling such that:
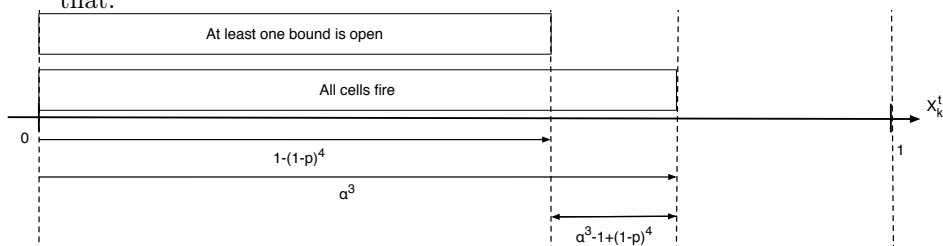
The probability that all cells fire is α^3 and the probability that at least one bound is open is $1-(1-p)^4$. Since $\alpha \geqslant \sqrt[3]{1-(1-p)^4}$, the three cells always fire when at least one bond is open in this coupling.

Thus as soon as one edge ending to site of $\hat{C}^{t+1}$ is open, the corresponding cell at time $t+1$ is active. Thus the correspondence criterion stays true at time $t+1$. Since the correspondence criterion is true at time 0, it will recursively stay true for all t. If $p > p_c$ then $\theta(p) > 0$ which means that for all $t \geqslant 0$ there exists a cell of height t which is in the open cluster with positive probability. Thus there exists an active cell in c^t for all $t \geqslant 0$, with positive probability. Then if $\alpha \geqslant \sqrt[3]{1-(1-p_c)^4}$ then $P_\alpha(c^{init}) < 1$. $\square$

Theorem 2 and theorem 3 yields the theorem 1.

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