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Midpoints and exact points of some algebraic functions in floating-point arithmetic

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In decimal128 arithmetic, the function $1/\sqrt{x^2 + y^2}$ has two midpoints in the range $[10^{-34}, 10^{-33})$, denoted by z_1 and z_2 . The pairs of floating-point numbers x and y such that $1/\sqrt{x^2 + y^2}$ equals z_1 or z_2 are listed below.

$$z_1 = 1.7763568394002504646778106689453125 \cdot 10^{-34}$$

$$z_2 = 8.8817841970012523233890533447265625 \cdot 10^{-34}$$

| x | y | z |
|--------------------------------------|-------------------------------------|-------|
| 5404319552844595200000000000000000 | 1576259869579673600000000000000000 | z_1 |
| 4503599627370496000000000000000000 | 3377699720527872000000000000000000 | |
| 5269211564023480320000000000000000 | 1981583836043018240000000000000000 | |
| 4746794007248502784000000000000000 | 3026418949592973312000000000000000 | |
| 42344645136388351590400000000000000 | 3709524941072530145280000000000000 | |
| 55082986610413252116480000000000000 | 1161856646267550040064000000000000 | |
| 51037529165935901933568000000000000 | 2375493879610755094937600000000000 | |
| 49626468536447581919641600000000000 | 2657706005508419097722880000000000 | |
| 40199832979566105169231872000000000 | 39409388843086146275639296000000000 | |
| 490889665236412098719986483200000000 | 27557210784319340005283594240000000 | |
| 51499148541640197927426064384000000 | 22736846748321363894428762112000000 | |
| 54841426882304976678598108774400000 | 12710011726327027640912628940800000 | |
| 43072865510444608119888968417280000 | 36247134470047764758330909655040000 | |
| 4685765143417034444156658442567680 | 3120075323841344966276075497717760 | |
| 5307519345123296639514855463714816 | 1876566920428820575559681455423488 | |
| 5613286575554586214400000000000000 | 4269412446747230208000000000000000 | |
| 55647410862208580122050560000000000 | 8514233077801196370001920000000000 | |
| 55805499689504571900769075200000000 | 7407611286729253918972313600000000 | |
| 5620657309038434535090972052684800 | 315398826977144693473134667366400 | |
| 5629499534213120000000000000000000 | 0 | |
| 1080863910568919040000000000000000 | 3152519739159347200000000000000000 | z_2 |
| 10538423128046960640000000000000000 | 3963167672086036480000000000000000 | |
| 11016597322082650423296000000000000 | 2323713292535100080128000000000000 | |
| 10207505833187180386713600000000000 | 4750987759221510189875200000000000 | |
| 11129482172441716024410112000000000 | 17028466155602392740003840000000000 | |
| 111610999379009143801538150400000000 | 14815222573458507837944627200000000 | |
| 10299829708328039585485212876800000 | 4547369349664272778885752422400000 | |
| 1096828537646099533571962175488000 | 254200234526540552818252578816000 | |
| 11226573151109172428800000000000000 | 8538824893494460416000000000000000 | |
| 1124131461807686907018194410536960 | 63079765395428938694626933473280 | |
| 9007199254740992000000000000000000 | 6755399441055744000000000000000000 | |
| 9493588014497005568000000000000000 | 6052837899185946624000000000000000 | |
| 8468929027277670318080000000000000 | 7419049882145060290560000000000000 | |
| 99252937072895163839283200000000000 | 53154120110168381954457600000000000 | |
| 80399665959132210338463744000000000 | 7881877768617229255127859200000000 | |
| 98177933047282419743997296640000000 | 55114421568638680010567188480000000 | |
| 861457310208892162397779368345600 | 724942689400955295166618193100800 | |
| 937153028683406888831331688513536 | 624015064768268993255215099543552 | |
| 11258999068426240000000000000000000 | 0 | |