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# Midpoints and exact points of some algebraic functions in floating-point arithmetic

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In decimal128 arithmetic, the function  $1/\sqrt{x^2 + y^2}$  has two midpoints in the range  $[10^{-34}, 10^{-33})$ , denoted by  $z_1$  and  $z_2$ . The pairs of floating-point numbers  $x$  and  $y$  such that  $1/\sqrt{x^2 + y^2}$  equals  $z_1$  or  $z_2$  are listed below.

$$z_1 = 1.7763568394002504646778106689453125 \cdot 10^{-34}$$

$$z_2 = 8.8817841970012523233890533447265625 \cdot 10^{-34}$$

$x$	$y$	$z$
54043195528445952000000000000000	15762598695796736000000000000000	$z_1$
45035996273704960000000000000000	33776997205278720000000000000000	
52692115640234803200000000000000	19815838360430182400000000000000	
47467940072485027840000000000000	30264189495929733120000000000000	
423446451363883515904000000000000	37095249410725301452800000000000	
550829866104132521164800000000000	11618566462675500400640000000000	
510375291659359019335680000000000	237549387961075509493760000000000	
496264685364475819196416000000000	265770600550841909772288000000000	
401998329795661051692318720000000	394093888430861462756392960000000	
490889665236412098719986483200000	275572107843193400052835942400000	
514991485416401979274260643840000	227368467483213638944287621120000	
548414268823049766785981087744000	127100117263270276409126289408000	
430728655104446081198889684172800	362471344700477647583309096550400	
468576514341703444415665844256768	312007532384134496627607549771776	
5307519345123296639514855463714816	1876566920428820575559681455423488	
56132865755545862144000000000000	4269412446747230208000000000000	
556474108622085801220505600000000	851423307780119637000192000000000	
558054996895045719007690752000000	74076112867292539189723136000000	
562065730903843453509097205268480	315398826977144693473134667366400	
5629499534213120000000000000000	0	
10808639105689190400000000000000	3152519739159347200000000000000	$z_2$
10538423128046960640000000000000	3963167672086036480000000000000	
11016597322082650423296000000000	2323713292535100080128000000000	
10207505833187180386713600000000	4750987759221510189875200000000	
11129482172441716024410112000000	1702846615560239274000384000000	
111610999379009143801538150400000	14815222573458507837944627200000	
102998297083280395854852128768000	45473693496642727788857524224000	
109682853764609953357196217548800	25420023452654055281825257881600	
11226573151109172428800000000000	853882489349446041600000000000	
112413146180768690701819441053696	63079765395428938694626933473280	
9007199254740992000000000000000	6755399441055744000000000000000	
9493588014497005568000000000000	6052837899185946624000000000000	
8468929027277670318080000000000	7419049882145060290560000000000	
9925293707289516383928320000000	5315412011016838195445760000000	
8039966595913221033846374400000	7881877768617229255127859200000	
98177933047282419743997296640000	55114421568638680010567188480000	
86145731020889216239777936834560	72494268940095529516661819310080	
937153028683406888831331688513536	624015064768268993255215099543552	
1125899906842624000000000000000	0	