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Midpoints and exact points of some algebraic functions in floating-point arithmetic

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In decimal128 arithmetic, the function $1/\sqrt{x^2 + y^2}$ has two midpoints in the range $[10^{-34}, 10^{-33})$, denoted by z_1 and z_2 . The pairs of floating-point numbers x and y such that $1/\sqrt{x^2 + y^2}$ equals z_1 or z_2 are listed below.

$$z_1 = 1.7763568394002504646778106689453125 \cdot 10^{-34}$$

$$z_2 = 8.8817841970012523233890533447265625 \cdot 10^{-34}$$

x	y	z
5404319552844595200000000000000000	1576259869579673600000000000000000	z_1
4503599627370496000000000000000000	3377699720527872000000000000000000	
5269211564023480320000000000000000	1981583836043018240000000000000000	
4746794007248502784000000000000000	3026418949592973312000000000000000	
42344645136388351590400000000000000	3709524941072530145280000000000000	
5508298661041325211648000000000000	1161856646267550040064000000000000	
5103752916593590193356800000000000	2375493879610755094937600000000000	
4962646853644758191964160000000000	2657706005508419097722880000000000	
4019983297956610516923187200000000	3940938884308614627563929600000000	
4908896652364120987199864832000000	2755721078431934000528359424000000	
5149914854164019792742606438400000	2273684674832136389442876211200000	
5484142688230497667859810877440000	1271001172632702764091262894080000	
4307286551044460811988896841728000	3624713447004776475833090965504000	
4685765143417034444156658442567680	3120075323841344966276075497717760	
5307519345123296639514855463714816	1876566920428820575559681455423488	
5613286575554586214400000000000000	4269412446747230208000000000000000	
5564741086220858012205056000000000	8514233077801196370001920000000000	
5580549968950457190076907520000000	740761128672925391897231360000000	
5620657309038434535090972052684800	315398826977144693473134667366400	
5629499534213120000000000000000000	0	
1080863910568919040000000000000000	3152519739159347200000000000000000	z_2
1053842312804696064000000000000000	3963167672086036480000000000000000	
1101659732208265042329600000000000	2323713292535100080128000000000000	
1020750583318718038671360000000000	4750987759221510189875200000000000	
1112948217244171602441011200000000	1702846615560239274000384000000000	
1116109993790091438015381504000000	1481522257345850783794462720000000	
1029982970832803958548521287680000	454736934966427277888575242240000	
1096828537646099533571962175488000	254200234526540552818252578816000	
1122657315110917242880000000000000	8538824893494460416000000000000000	
1124131461807686907018194410536960	63079765395428938694626933473280	
9007199254740992000000000000000000	6755399441055744000000000000000000	
9493588014497005568000000000000000	6052837899185946624000000000000000	
8468929027277670318080000000000000	7419049882145060290560000000000000	
9925293707289516383928320000000000	5315412011016838195445760000000000	
8039966595913221033846374400000000	7881877768617229255127859200000000	
9817793304728241974399729664000000	5511442156863868001056718848000000	
861457310208892162397779368345600	724942689400955295166618193100800	
937153028683406888831331688513536	624015064768268993255215099543552	
1125899906842624000000000000000000	0	