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# Midpoints and exact points of some algebraic functions in floating-point arithmetic

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## To cite this version:

Claude-Pierre Jeannerod, Nicolas Louvet, Jean-Michel Muller, Adrien Panhaleux. Midpoints and exact points of some algebraic functions in floating-point arithmetic. 2010. ensl-00409366v2

**HAL Id: ensl-00409366**

**https://ens-lyon.hal.science/ensl-00409366v2**

Preprint submitted on 22 Mar 2010 (v2), last revised 4 Jan 2011 (v3)

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In decimal128 arithmetic, the function $1/\sqrt{x^2+y^2}$ has two midpoints in the range $[10^{-34}, 10^{-33})$, denoted by $z_1$ and $z_2$. The pairs of floating-point numbers $x$ and $y$ such that $1/\sqrt{x^2+y^2}$ equals $z_1$ or $z_2$ are listed below.

$$z_1 = 1.776356839400250464677810668945312 5 \cdot 10^{-34}$$
$$z_2 = 8.881784197001252323389053344726562 5 \cdot 10^{-34}$$

| $x$ | $y$ | $z$ |
|---|---|---|
| 5404319552844595200000000000000000 | 1576259869579673600000000000000000 | $z_1$ |
| 4503599627370496000000000000000000 | 3377699720527872000000000000000000 | |
| 5269211564023480320000000000000000 | 1981583836043018240000000000000000 | |
| 4746794007248502784000000000000000 | 3026418949592973312000000000000000 | |
| 4234464513638835159040000000000000 | 3709524941072530145280000000000000 | |
| 5508298661041325211648000000000000 | 1161856646267550040064000000000000 | |
| 5103752916593590193356800000000000 | 2375493879610755094937600000000000 | |
| 4962646853644758191964160000000000 | 2657706005508419097722880000000000 | |
| 4019983297956610516923187200000000 | 3940938884308614627563929600000000 | |
| 4908896652364120987199864832000000 | 2755721078431934000528359424000000 | |
| 5149914854164019792742606438400000 | 2273684674832136389442876211200000 | |
| 5484142688230497667859810877440000 | 1271001172632702764091262894080000 | |
| 4307286551044460811988896841728000 | 3624713447004776475833090965504000 | |
| 4685765143417034444156658442567680 | 3120075323841344966276075497717760 | |
| 5307519345123296639514855463714816 | 1876566920428820575559681455423488 | |
| 5613286575554586214400000000000000 | 4269412446747230208000000000000000 | |
| 5564741086220858012205056000000000 | 8514233077801196370001920000000000 | |
| 5580549968950457190076907520000000 | 7407611286729253918972313600000000 | |
| 5620657309038434535090972052684800 | 3153988269771446934731346673664 00 | |
| 5629499534213120000000000000000000 | 0 | |
| 1080863910568919040000000000000000 | 3152519739159347200000000000000000 | $z_2$ |
| 1053842312804696064000000000000000 | 3963167672086036480000000000000000 | |
| 1101659732208265042329600000000000 | 2323713292535100080128000000000000 | |
| 1020750583318718038671360000000000 | 4750987759221510189875200000000000 | |
| 1112948217244171602441011200000000 | 1702846615560239274000384000000000 | |
| 1116109993790091438015381504000000 | 1481522257345850783794462720000000 | |
| 1029982970832803958548521287680000 | 4547369349664272778885752422400000 | |
| 1096828537646099533571962175488000 | 2542002345265405528182525788160000 | |
| 1122657315110917242880000000000000 | 8538824893494460416000000000000000 | |
| 1124131461807686907018194410536960 | 6307976539542893869462693347328 0 | |
| 9007199254740992000000000000000000 | 6755399441055744000000000000000000 | |
| 9493588014497005568000000000000000 | 6052837899185946624000000000000000 | |
| 8468929027277670318080000000000000 | 7419049882145060290560000000000000 | |
| 9925293707289516383928320000000000 | 5315412011016838195445760000000000 | |
| 8039966595913221033846374400000000 | 7881877768617229255127859200000000 | |
| 9817793304728241974399729664000000 | 5511442156863868001056718848000000 | |
| 8614573102088921623977793683456000 | 7249426894009552951666181931008000 | |
| 9371530286834068888313316885135360 | 6240150647682689932552150995435520 | |
| 1125899906842624000000000000000000 | 0 | |