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## Experimental study of work exchange with a granular gas: the viewpoint of the Fluctuation Theorem.

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**Abstract** – This article reports on an experimental study of the fluctuations of energy flux between a granular gas and a small driven harmonic oscillator. The DC-motor driving this system is used simultaneously as actuator and probe. The statistics of work fluctuations at controlled forcing, between the motor and the gas are examined from the viewpoint of the Fluctuation Theorem. A characteristic energy  $E_c$  of the granular gas, is obtained from this relation between the probabilities of an event and its reversal.

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**Introduction.** – Often, the principle of probing is essentially to impose a perturbation to a system, and measure its response. This perturbation must be small enough not to affect the state of the system under scrutiny. Together with the Fluctuation Theorem (FT), this principle is used in the present study to probe the disordered motion of a granular gas.

In addition to the power needed to keep the granular gas in a Non Equilibrium Steady State (NESS), (*i.e.* off-setting the mean dissipation), a small probing power is imposed. The response to this perturbation is characteristic of the granular gas disordered *state*. More precisely here, it is the product of the forcing perturbation and the velocity response, *i.e.* the power perturbation, that carries information on the system: its statistical properties will be considered in order to characterise the disordered granular gas *state*. This holds as long as the perturbation is small enough not to effects the dynamics of the gas.

The measurement (excitation-response), is performed by means of a driven harmonic oscillator coupled to the granular gas. The Fluctuation Theorem (FT) is used to compare the work given and received from the granular gas by the harmonic oscillator.

The power provided (work flux) can be related to the rate of entropy production  $\sigma$ . The FT states a relation between the probability of events of positive  $\sigma$ , and that of equal but negative entropy production rate. The FT states that the ratio of these probabilities simply increases as the ex-

ponential of  $\sigma$  [1]. It therefore quantifies the failure of detailed balance, for a large number of degrees of freedom chaotic dynamical system.

This experimental study applies an original and very simple probing principle, to show that FT actually holds in a stationary granular gas, and measure a *characteristic energy*  $E_c$ .

Rather few experiments invoking the FT in granular gases have been actually done. Most studies are numerical [2], or theoretical [3]. Strictly speaking, the only experimental study of a granular gas with the FT is that of Feitosa and Menon [4]. In a 2D vibrated granular material, they measured by video tracking the momentum flux of particles in and out of a sub-volume, and used the FT to define an effective temperature of the medium. The interpretation of these results is difficult, and still questioned [3].

In a distinct context, D'Anna *et. al.* performed rheology in dense granular fluids using a driven torsion pendulum. Thanks to an out-of-equilibrium extension of the Fluctuation-Dissipation Theorem, they defined an effective temperature [5].

The present article describes an original study of a dilute 3D granular gas, also using a driven torsion pendulum. In a simpler but reliable manner, it show how to measure an energy  $E_c$ , characteristic of the particles disordered motion, thanks to the FT.

The equation of motion of the harmonic oscillator writes

as the following:

$$M\ddot{\theta} + \gamma\dot{\theta} + k\theta = \Gamma(t) + \eta(t), \quad (1)$$

where  $\theta$  is the angle of torsion of the pendulum, and dots stand for time derivatives.  $M$ ,  $\gamma$ ,  $k$  are respectively moment of inertia, viscous friction coefficient, and spring constant. A sine torque  $\Gamma(t)$  is imposed from outside. The last term  $\eta(t)$  represents the coupling with the NESS granular gas bath. In the framework of Langevin equation description, it represents the momentum transfer rate at each shock from the beads.

When the FT is expressed in terms of the work variation rate  $\dot{w}(t) = \Gamma\dot{\theta}$ , the mechanical power transmitted during a time-lag  $\tau$  to the gas is expressed as:  $\dot{w}_\tau(t) = \frac{1}{\tau} \int_\tau \dot{w}(t') dt'$ . It relates the probabilities of giving a power  $\dot{w}_\tau$  to the gas and the probability of receiving the same amount from it. It states that this ratio increases exponentially with the coarse-grained work  $\tau\dot{w}_\tau$ :

$$\frac{\Pi(\dot{w}_\tau)}{\Pi(-\dot{w}_\tau)} = e^{\tau\dot{w}_\tau/E_c}, \quad (2)$$

for asymptotically large times  $\tau$ .  $\Pi$  is the probability, and the coefficient  $E_c$  is a *characteristic energy* (possibly linked to the mean kinetic energy of the gas).

The principle of the measurement and the experimental set-up are described in the next section. The results obtained are detailed in the following section. The last section is devoted to a brief discussion of these results and openings for future studies.

**Measurement's principle.** – Fig. 1 sketches the experimental set-up. The granular gas is composed of a few hundreds of 3 mm diameter stainless steel beads, contained in a vibrating vessel. The vessel is aluminum made, 5 cm diameter, 6 cm deep, which inside bottom is slightly cone-shaped to favour horizontal momentum transfer (angle=10°). The electromechanical vibrator is driven by a sine generator, via a Kepco current amplifier. An accelerometer fixed on the vibrating vessel gives the vertical acceleration: between  $41 \text{ ms}^{-2}$  and  $60 \text{ ms}^{-2}$ , at a frequency  $f_{\text{exc.}} = 40 \text{ Hz}$ . A small DC-motor is simultaneously used as actuator and sensor. It is a regular permanent magnet, brushed DC-motor, of relatively small size (25 mm diameter). A plastic blade of approximately 20 x 20 mm is fixed on the axis of the motor, with a spiral torsion spring. This system motor + blade + spring forms a harmonic oscillator. Its resonance frequency is a few hundreds of Hz, in any case higher than any frequency of the signal of interest. The motor is fixed on a cover closing the vessel, which prevents the beads from hopping off the vessel.

The principle of the measurement is the following. A DC-motor can be run as a motor as well as a generator. As a generator, the induced voltage is proportional to the angular velocity:  $e = \alpha\dot{\theta}$ . As a motor, the torque is proportional to the current supplied  $I$ :  $\Gamma = \alpha I$ . Notice that

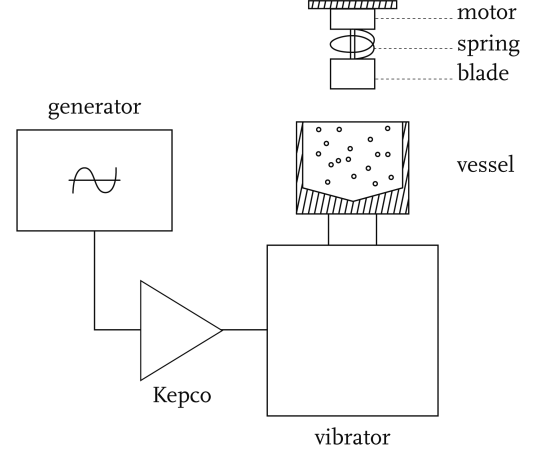


Fig. 1: The mechanical system is composed of a vibrating vessel with its driving, and the probing harmonic oscillator, pulled out for clarity.

both relations involve the same proportionality coefficient, which depends on physical parameters of the motor itself. The mechanical work produced by the motor against the granular gas per unit time is simply:  $\dot{w} = \Gamma\dot{\theta} = e I$ . The motor's command and measurement set-up are sketched in fig. 2. The harmonic oscillator is driven by an AC current, supplied through a  $R = 1 \text{ k}\Omega$  resistor by a sine voltage generator at  $f_e = 13 \text{ Hz}$  frequency. As  $R$  is

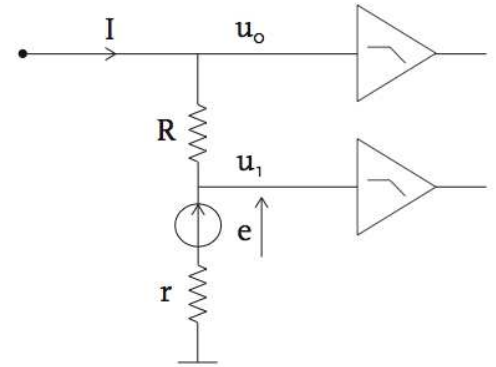


Fig. 2: The electrical sketch of the motor's command, and measurement set-up, that is to say the amplifiers-filters stage. The motor is represented by the induced voltage 'source'  $e$  and its internal resistance  $r$ .

large enough, the motor is current-driven (see time-series in fig. 3). At such a low frequency, iron losses in the motor are negligible. The voltages  $u_0$  and  $u_1$  are recorded by a 16 bits simultaneous acquisition board at frequency  $f_s = 1024$  Hz, after amplification and anti-alias filtering at 512 Hz. The results reported here are obtained from one hour-long recordings. The current  $I(t) = (u_0 - u_1)/R$  and the induction voltage  $e(t) = u_1 - rI(t)$  are measured,  $r$  being the internal resistance of the motor ( $r \simeq 5.8 \Omega$ ). The power  $\dot{w}(t)$  produced by the motor at 13 Hz frequency is completely transferred to the granular gas. Although

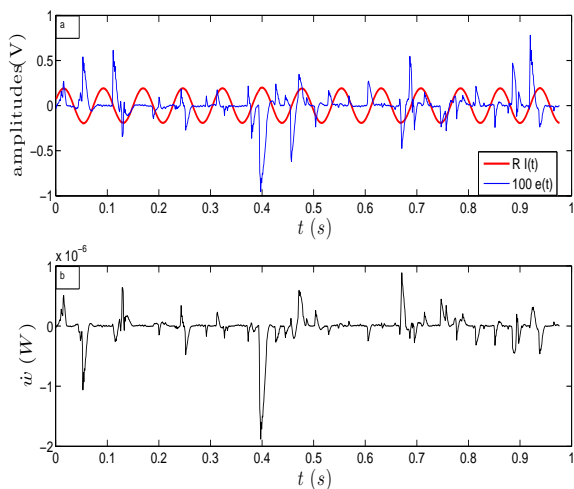


Fig. 3: a) Time-series of the imposed sine current ( $I \simeq 200 \mu A$ ), and the induced voltage. For homogeneity and comparison, both have been plotted in Volts:  $R I(t)$ , and for readability,  $e$  has been enlarged by a factor 100. b) In the same time-scale, the corresponding power is plotted. The mean power given is  $\langle \dot{w} \rangle \simeq 7.3$  pW. The acceleration is  $a \simeq 52$  ms $^{-2}$ .

$I(t)$  is a sine,  $e(t)$  fluctuates strongly because of the beads collisions on the blade. The power  $\dot{w} = eI$  shows large fluctuations, negative whenever  $I$  and  $e$  have opposite sign (see fig. 3). In such case, the reservoir is giving work to the blade-motor system. That corresponds to an instantaneous decrease of the entropy of the total system.

Looking carefully at the time series, one cannot see in the fluctuations of the voltage any trace of the sine excitation. This is confirmed by the power spectrum on fig. 4: the peak of the response at 13 Hz is hardly visible over the large-band spectrum. At the same time, the fluctuations of power are widely distributed positively and negatively.

**Results.** – Histograms of the power  $\dot{w}_\tau$  transmitted to the gas over several time-lags  $\tau$  are calculated:  $\dot{w}_\tau(t) = \frac{1}{\tau} \int_\tau e(t')I(t')dt'$ . Time-lags are chosen as integer multiples of the excitation period:  $\tau = n/f_e$ . Convenient values are chosen as  $n = 20, 40, 60, 80, 100$ , that is  $\tau \simeq 1.5$  s, 3.1 s, 4.6 s, 6.1 s, 7.7 s. Corresponding histograms are shown in fig. 5. Note that here and in the following, the whole statistics is shown, including the less

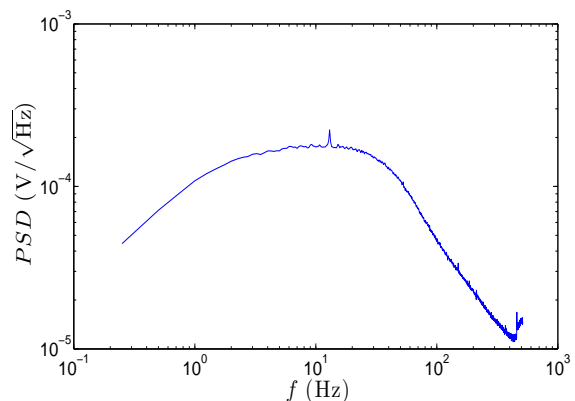


Fig. 4: For the same as the previous measurement, power spectral densities of  $e$  is plotted, showing that the induced voltage (velocity) is broad-band noise. The excitation is visible at 13 Hz.

probable events, accounting for noisy tails. The wider histograms correspond to the smallest  $\tau$ .

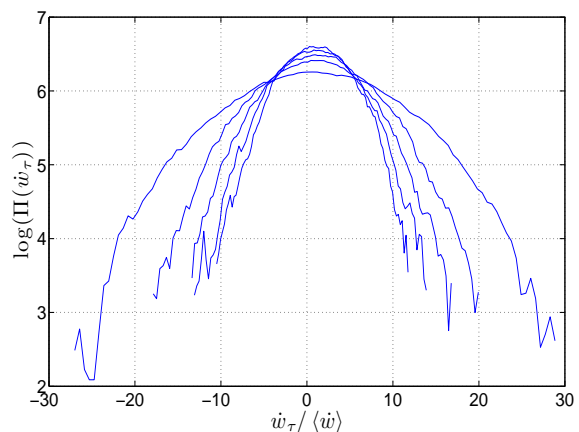


Fig. 5: Histograms of the power injected to the gas, time-averaged over  $\tau \simeq 1.5$  s, 3.1 s, 4.6 s, 6.1 s, 7.7 s, *i.e.* 20 to 100 periods of excitation.  $\dot{w}_\tau$  is normalised by the mean value  $\langle \dot{w} \rangle \simeq 6$   $10^{-8}$  W.

A few remarks before going further:

- The mean value is slightly positive, as the motor gives work to the bath on the average. However, fluctuations are large enough to present negative values. This is the reason why the current is chosen such that fluctuations are rather large if compared to the mean power.
- The histograms are symmetric with respect to the mean value.
- The convergence to a Gaussian as  $\tau$  increases is actually very slow, because the process  $\dot{w}$  has a long time memory.

Starting from those histograms, eq. 2 is examined, plotting the so-called *symmetry function*:  $\frac{1}{\tau} \log \left( \frac{\Pi(\dot{w}_\tau)}{\Pi(-\dot{w}_\tau)} \right)$  against  $\dot{w}_\tau$ . One can see in fig. 6 that these curves are approximately linear, at least for moderate values of  $\dot{w}_\tau$ , validating experimentally the prediction of the FT. In some cases however, curves are observed to be slightly bent downward for large values of  $\dot{w}_\tau$ . This is due to  $\tau$  being too small.

In order to test eq. 2, the slope of the symmetry function is evaluated. To avoid the fitting being dominated by extreme but less relevant values, it is performed for each  $\tau$  only over amplitudes  $\dot{w}_\tau$  less than half of the maxima. This procedure is robust for all experimental configurations considered here (change of  $I$ ). The fitting

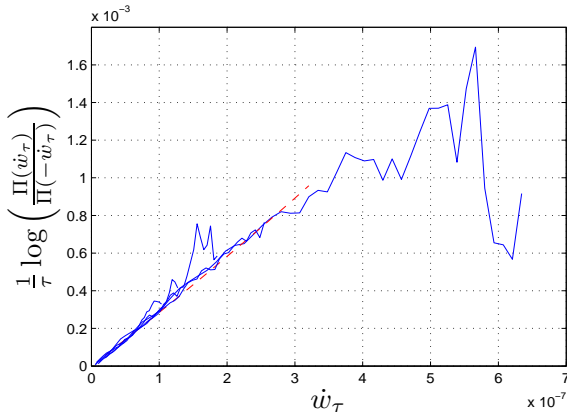


Fig. 6:  $\frac{1}{\tau} \log \left( \frac{\Pi(\dot{w}_\tau)}{\Pi(-\dot{w}_\tau)} \right)$  is plotted against  $\dot{w}_\tau$ , for the same 5 time-lags as on previous figure. A fitting is performed, shown only at the smallest  $\tau$  (dashed line).

parameter is the slope  $1/E_c$ , where  $E_c$  is called *characteristic energy*. It is evaluated for different values of the time-lag  $\tau$ . It converges toward a limit value at large  $\tau$ , proving experimentally that the FT holds (see fig. 7). For long times, however, negative events become rare, causing statistical uncertainty on  $E_c$ . This is the very reason why the parameters must be chosen such that  $\langle \dot{w} \rangle$  is small, and fluctuation are large.

This measurement is re-conducted for several values of the current  $I$ . The curves shown in fig. 7 collapse to a unique value  $E_c \simeq 5 \cdot 10^{-4} \text{ J}$ . Uncertainties are difficult to evaluate, but it is believed that the dispersion of the curves is mainly due to the fitting process, and to statistical limitations.

The collapse of the curves in fig. 7, obtained at different  $I$  means that within a certain range of current amplitudes,  $E_c$  does not depend on the blade-motor system and its excitation. It is therefore a characteristic of the granular gas solely. In other words, this externally driven blade-motor system actually qualifies as a probe.

The range of possible currents is however bounded. If

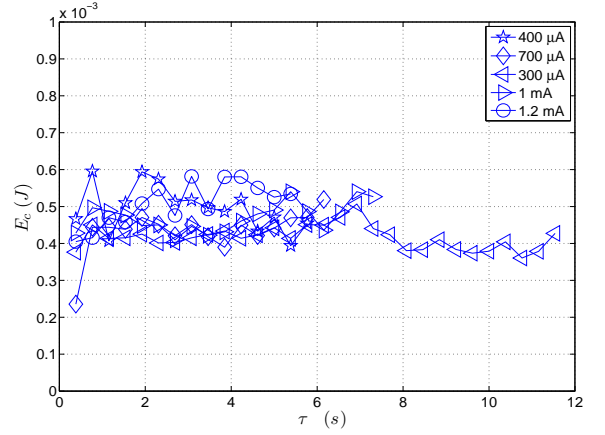


Fig. 7:  $E_c$  against  $\tau$ , for several values of the excitation current, but the same acceleration:  $a = 41 \text{ ms}^{-2}$ .

$I$  is too large, velocity simply follow the torque and no negative events are observed:  $E_c$  cannot be measured. If  $I$  is too small, the histograms are almost symmetric, causing a poor resolution of  $E_c$ . The range respecting this compromise is in practice large enough for measurement. Moreover, it must be noted that the measurement is a very small perturbation as it involves mean powers of the order of a few  $10^{-8} \text{ W}$ , which is extremely small in such a macroscopic experiment !

What can the energy  $E_c$  be compared to ? As the blade is set about a cm above the bottom of the vessel, the energy transmitted from the beads can naïvely be related to the energy of the particles jumping at this height. In such crude comparison, if they are given an acceleration  $a$ , the mean energy they gain is:  $E \simeq m a h$ , with the mass of a bead  $m = 0.1 \text{ g}$ , the acceleration  $a \simeq 41 \text{ ms}^{-2}$  (for the measurement presented in fig. 7), and a height of, say,  $h \simeq 2 \text{ cm}$ . Therefore,  $E \simeq m a h \simeq 10^{-4} \text{ J}$ . The order of magnitude is satisfactory, considering that several beads might hit the blade together.

The accuracy of the measurements of  $E_c$  is not sufficient presently to investigate its dependance in  $a$ . The difficulty comes from the hardness to measure the asymptotic  $E_c$ . In other terms, the range of coarse-graining to perform is the result of a delicate compromise.  $\tau$  must be large enough to overcome the transitory and give an asymptotic value of the slope, however still allow enough negative fluctuations to measure  $E_c$  at all !

The constraints might be more difficult to satisfy all together for a wide range of acceleration.

**Conclusion and perspective.** — The system studied here is a relatively small harmonic oscillator, embedded in a granular gas a probe to characterise it. It is composed of a DC-motor with a blade and a torsion spring on its axis. The blade is embedded in a large number of beads maintained in a gaseous state by an external power supply

$P_{\text{exc.}}$  from a vibrator. The granular gas has a extremely large number of degrees of freedom, which guaranties an efficient chaotic loss of memory. It forms a NESS bath, to which the system *thermalises*. It mimics quite well a real gas, however with the following main differences: the number of particles is not extremely large (fluctuations are important), the collisions are dissipative (a power supply is necessary to compensate this dissipation).

The DC-motor driving the small harmonic oscillator is used as actuator and sensor, to probe the granular gas. Some relations in a DC-motor between mechanical and electrical quantities are such that no calibration is needed to measure the power delivered to the bath. This property is fortunate, as calibration is often a source of errors.

Through this device, a small energy flux  $\dot{w}$  is injected into the granular gas at controlled torque. This perturbation power is shown to verify the FT.

An output quantity, a *characteristic energy*  $E_c$ , is measured and shown not to depend on the imposed torque of the motor within a certain range. Therefore, one conclude that  $E_c$  is characteristic of the granular gas, *i.e.* the small system is actually a probing system.

The measurement of  $E_c$  at the present stage is not precise enough to find a relation with the acceleration  $a$  for different  $P_{\text{exc.}}$ , although it is expectedly related. Improvements of the set-up are in progress.

One interesting question is whether  $E_c$  obtained via the FT in the present study is the same or different as that obtained using the fluctuations of the power supply  $P_{\text{exc.}}$  (like in most of the preceding studies, for instance in [2]). It is possible to measure with arbitrary long statistics negative fluctuations of injected power, for a granular gas as far as desired from equilibrium. This key point was discussed by Zamponi [6] as a limiting benchmark for experiments. Applying the FT to a small probing power instead of the power supplying the granular gas is a biased opinion, which makes this difference.

It will be interesting in the future to understand the physical significance of  $E_c$ . The FT, as often considered, gives an interpretation of the characteristic energy in terms of an effective (non-equilibrium) temperature:  $E_c \equiv k_B T_{\text{eff.}}$ . In the present case however, such interpretation deserves caution, as the forcing is non-Gaussian, the granular gas has an irreversible dynamics, and is intrinsically dissipative. Interpretation of these measurements remains to be done.

\* \* \*

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## REFERENCES

- [1] EVANS D. J., and COHEN E. G. D., and MORRIS G. P., *Phys. Rev. Lett.*, **71** (1993) 2401, GALLAVOTTI G., and COHEN E.D.G., *Phys. Rev. Lett.*, **74** (1995) 2694-2697
- [2] AUMAÎTRE S. and FARAGO J. and FAUVE S. and MC NAMARA S., *Eur. Phys. J. B*, **42** (2004) 255-261
- [3] PUGLISI A. and VISCO P. and BARRAT A. and TRIZAC E. and VAN WIJLAND F., *Phys.Rev. Lett.*, **95** (2005) 110202, and references therein,
- [4] FEITOSA K. and MENON N., *Phys. Rev. Lett.*, **92** (2004) 164301.
- [5] DANNA G. and MAYOR P. and BARRAT A. and LORETO V. and NORI F., *Nature*, **424** (2003) 909912.
- [6] ZAMPONI F., *J. Stat. Mech.*, **2007** (P02008)