

Experimental study of work exchange with a granular gas: the viewpoint of the Fluctuation Theorem.

Antoine Naert

▶ To cite this version:

Antoine Naert. Experimental study of work exchange with a granular gas: the viewpoint of the Fluctuation Theorem.. 2011. ensl-00610684v2

HAL Id: ensl-00610684 https://ens-lyon.hal.science/ensl-00610684v2

Preprint submitted on 27 Sep 2011 (v2), last revised 9 Dec 2011 (v4)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers. L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Experimental study of work exchange with a granular gas: the viewpoint of the Fluctuation Theorem.

Antoine Naert.

Laboratoire de Physique de l'École Normale Supérieure de Lyon, Université de Lyon, CNRS UMR 5672, 46 Allée d'Italie, 69364 Lyon cedex 7, France.

PACS 05.70.Ln – Nonequilibrium and irreversible thermodynamics PACS 05.40.-a – Fluctuation phenomena, random processes, noise, and Brownian motion PACS 45.70.Cc – granular systems

Abstract – This article reports on an experimental study of the fluctuations of energy flux between a granular gas and a small driven harmonic oscillator. The DC-motor driving this system is used simultaneously as actuator and probe. The statistics of work fluctuations at controlled forcing, between the motor and the gas are examined from the viewpoint of the Fluctuation Theorem. A characteristic energy E_c of the granular gas, is obtained from this relation between the probabilities of an event and its reversal.

Introduction. — Often, the principe of probing is essentially to measure the response of a system to an excitation imposed from outside. This excitation must be small enough not to perturb the state of the system under scrutiny. Together with the Fluctuation Theorem (FT), this principle is used in the present study to probe the disordered motion of a granular gas.

In addition to the power needed to keep the granular gas in a Non Equilibrium Steady State (NESS), (i.e. off-setting the mean dissipation), a small probing power is imposed. The response to this excitation is characteristic of the granular gas disordered state. More precisely here, it is the product of the forcing perturbation and the velocity response, i.e. the power perturbation, that carries information on the system: its statistical properties will be considered in order to characterise the disordered granular gas NESS.

The measurement (excitation-response), is performed by means of a driven harmonic oscillator *coupled* to the granular gas. The Fluctuation Theorem (FT) is used to compare the work given and received from the granular gas by the harmonic oscillator. It is the Steady State Fluctuation Theorem (SSFT) that is considered all along this article, *i.e.* a relation holding in the limit of asymptotically large times. No further mention of this distinction will be given later

The power provided (work flux) can be related to the rate of entropy production σ . The FT states a relation be-

tween the probability of events of positive σ , and that of equal but negative entropy production rate. It states that the ratio of these probabilities simply increases as the exponential of σ [1]. It therefore quantifies the failure of detailed balance, for a large number of degrees of freedom chaotic dynamical system.

This experimental study applies an original and very simple probing principle, to show that FT seems to holds in a stationary granular gas, and measure a *characteristic energy* E_c . It is a contribution to the very few experimental studies of the different forms of the FT. (For a nice review, see [2].)

Rather few experiments invoking the FT have been actually done in granular gases particularly. Most studies are numerical [3], or theoretical [4]. Strictly speaking, the only experimental study of a granular gas with the FT is that of Feitosa and Menon [5]. In a 2D vibrated granular material, they measured by video tracking the momentum flux of particles in and out of a sub-volume, and used the FT to define an effective temperature of the medium. The interpretation of these results is difficult, and still questioned [4].

In a distinct context, D'Anna et. al. performed rheology in dense granular fluids using a driven torsion pendulum. Thanks to an out-of-equilibrium extension of the Fluctuation-Dissipation Theorem, they defined an effective temperature [6].

The present article describes an original study of a dilute

3D granular gas, also using a driven torsion pendulum. In a simpler but reliable manner, it show how to measure an energy E_c , characteristic of the particles disordered motion, thanks to the FT.

The equation of motion of the harmonic oscillator writes as the following:

$$M\ddot{\theta} + \gamma\dot{\theta} + k\theta = \Gamma(t) + \eta(t),$$
 (1)

where θ is the angle of torsion of the pendulum, and dots stand for time derivatives. M, γ, k are respectively moment of inertia, viscous friction coefficient, and spring constant. A sine torque $\Gamma(t)$ is imposed from outside. The last term $\eta(t)$ represents the coupling with the NESS granular gas bath. In the framework of Langevin equation description, it represents the momentum transfer rate at each shock from the beads.

When the FT is expressed in terms of the work variation rate $\dot{w}(t) = \Gamma \dot{\theta}$, the mechanical power transmitted during a time-lag τ to the gas is expressed as: $\dot{w}_{\tau}(t) = \frac{1}{\tau} \int_{\tau} \dot{w}(t') dt'$. It relates the probabilities of giving a power \dot{w}_{τ} to the gas and the probability of receiving the same amount from it. It states that this ratio increases exponentially with the coarse-grained work $\tau \dot{w}_{\tau}$:

$$\frac{\Pi(\dot{w}_{\tau})}{\Pi(-\dot{w}_{\tau})} = e^{\tau \dot{w}_{\tau}/E_c},\tag{2}$$

for asymptotically large times τ . Π is the probability, and the coefficient E_c is a *characteristic energy* (possibly linked to the mean kinetic energy of the gas).

The principle of the measurement and the experimental set-up are described in the next section. The results obtained are detailed in the following section. The last section is devoted to a brief discussion of these results and openings for future studies.

Measurement's principle. – Fig. 1 sketches the experimental set-up. The granular gas is composed of a few hundreds of 3 mm diameter stainless steel beads, contained in a vibrating vessel. The vessel is aluminum made, 5 cm diameter, 6 cm deep, its inside bottom is slightly cone-shaped to favour horizontal momentum transfer (angle=10°). (See fig. 1.) The electromechanical shaker is driven by a sine generator, via a Kepco current amplifier. An accelerometer fixed on the vibrating vessel measures the vertical acceleration: between $41 \,\mathrm{ms}^{-2}$ and $60 \,\mathrm{ms^{-2}}$, at a frequency $f_{\mathrm{exc.}} = 40 \,\mathrm{Hz}$. A small DC-motor is simultaneously used as actuator and sensor. It is a regular permanent magnet, brushed DC-motor, of relatively small size (25 mm diameter). A plastic blade of approximately 20 x 20 mm is fixed on the axis of the motor. A torsion spring is used to produce an elastic force on the motor axis. This system motor + blade + spring forms a harmonic oscillator. Its resonance frequency is a few hundreds of Hz, in any case higher than any frequency of the signal of interest. The motor is fixed on a cover closing the vessel, which prevents the beads from hopping off the

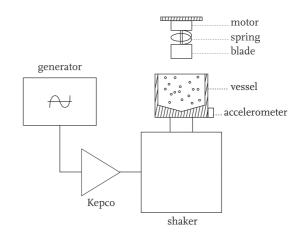


Fig. 1: The mechanical system is composed of a vibrating vessel with its driving, and the probing harmonic oscillator, pulled out for clarity.

vessel.

The principle of the measurement is the following. A DC-motor can be used as a motor as well as a generator. As a generator, the induced voltage is proportional to the angular velocity: $e = \alpha \dot{\theta}$. As a motor, the torque is proportional to the current supplied I: $\Gamma = \alpha I$. Notice that both relations involve the same proportionality coefficient, which depends on the physical parameters of the motor itself. The mechanical work produced by the motor against the granular gas per unit time is simply: $\dot{w} = \Gamma \dot{\theta} = e I$.

The motors command and measurement set-up are sketched in fig. 2. The harmonic oscillator is driven by an AC current, supplied through a $R=1\,k\Omega$ resistor by a sine voltage generator at $f_e=13\,\mathrm{Hz}$ frequency. As R is large enough, the motor is driven by a periodic current (see time-series in fig. 3). At such a low frequency, iron losses in the motor are negligible. The voltages u_0 and u_1 are recorded by a 16 bits simultaneous acquisition board at frequency $f_s=1024\,\mathrm{Hz}$. The amplificators adjusts the level of the signals to that of the A/N converter, and anti-alias filters at $512\,\mathrm{Hz}$. The results reported here are obtained from one hour-long recordings. The current $I(t)=(u_0-u_1)/R$ and the induction voltage $e(t)=u_1-r\,I(t)$ are measured, r being the internal resistance of the motor ($r\simeq5.8\,\Omega$).

The power $\dot{w}(t) = \Gamma \dot{\theta} = e I$ produced by the motor at 13 Hz frequency is completely transferred to the granulafr gas. Although I(t) is a sine, e(t) fluctuates strongly because of the beads collisions on the blade. The power $\dot{w} = e I$ shows large fluctuations, negative whenever I and e have opposite sign (see fig. 3). In such case, the reservoir

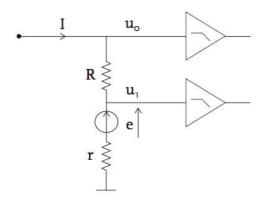


Fig. 2: The electrical sketch of the motor's command, and measurement set-up, that is to say the amplifiers-filters stage. The motor is represented by the induced voltage 'source' e and its internal resistance r.

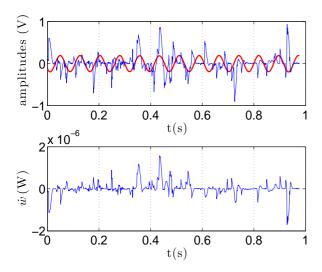


Fig. 3: Top: time-series of the imposed sine current ($I \simeq 200 \mu A$), and the induced voltage. For homogeneity and comparison, both have been plotted in Volts: R~I(t) (bold), and for readability, e has been enlarged by a factor 100 (thin). Bottom: in the same time-scale, the corresponding energy flux $\dot{w}(t)$ is plotted. The mean power given is $\langle \dot{w} \rangle \simeq 7.3~nW$. The acceleration is $a \simeq 56~{\rm ms}^{-2}$.

is giving work to the blade-motor system. The fluctuations of power are widely distributed positively and negatively. Looking carefully at the time series, one cannot see in the fluctuations of the voltage any trace of the sine excitation. One can see in fig. 4 the power spectral densities of the induced voltage e. It is broad band noise, reflecting that the momentum transfer from the granular gas is disordered: no periodic contribution is visible except a small contribution at $13\ Hz$ coming from the current.

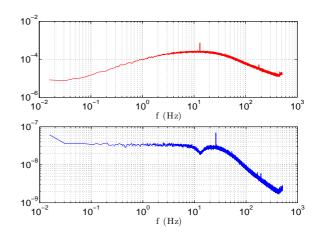


Fig. 4: For the same as the previous measurement, the power spectral densities of e is plotted on the top, showing that the induced voltage (velocity) is broad-band noise. The excitation is visible at $f_e = 13\,\mathrm{Hz}$. The bottom plot shows the power spectral density of the probing power \dot{w} .

Results. – Histograms of the power \dot{w}_{τ} transmitted to the gas over several time-lags τ are calculated: $\dot{w}_{\tau}(t) = \frac{1}{\tau} \int_{\tau} e(t') I(t') dt'$. Time-lags are chosen as integer multiples of the excitation period: $\tau = n/f_e$. Convenient values are chosen as n = 10, 30, 50, 70, 90. Corresponding histograms are shown in fig. 5. They are calculated over 50 bins. The wider histograms correspond to the smallest τ . Starting from those histograms, eq. 2

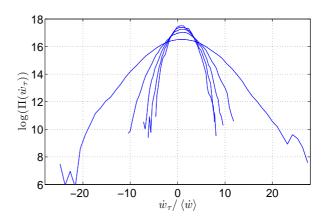


Fig. 5: Histograms of the power injected to the gas, time-averaged over $\tau \simeq 0.77\,s, 2.3\,s, 3.85\,s, 5.38\,s, 6.92\,s,$ *i.e.* 980 to $8.9\,10^3\,\tau_c$, where $\tau_c \simeq 7.8\,10^{-4}\,s.$ \dot{w}_{τ} is normalised by the mean value $\langle \dot{w} \rangle \simeq 5.3\,10^{-9}\,W.$

is examined, plotting against \dot{w}_{τ} the so-called asymmetry function: $\frac{1}{\tau}log\left(\frac{\Pi(\dot{w}_{\tau})}{\Pi(-\dot{w}_{\tau})}\right)$. One can see in fig. 6 that these curves are approximately linear, at least for moderate \dot{w}_{τ} , validating experimentally the prediction of FT. In some cases however, curves are observed to be slightly

bent downward for large values of \dot{w}_{τ} . This is due to τ being too small.

In order to test eq. 2, the slope of the asymmetry function is evaluated. To avoid the fitting being dominated by extreme but less relevant values, it is performed for each τ only over amplitudes \dot{w}_{τ} less than half of the maxima. A ratio slightly different than 1/2, like 1/3 or 2/3, gives the same results. With 1/2, this procedure is stable for all experimental configurations considered here (change of I, and acceleration of the beads). The fitting parameter is

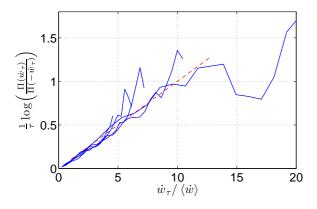


Fig. 6: The asymmetry function $\frac{1}{\tau}log\left(\frac{\Pi(\dot{w}_{\tau})}{\Pi(-\dot{w}_{\tau})}\right)$ is plotted against $\dot{w}_{\tau}/\left\langle\dot{w}_{\tau}\right\rangle$, for the same 5 time-lags as on previous figure. A fitting is performed, shown only at the smallest τ (dashed line).

the slope $1/E_c$, where E_c is called characteristic energy. It is evaluated for different values of the time-lag τ . It tends to converge toward a limit value called E_c , at large τ although with some scattering (see fig. 6 and fig. 7). For long times though, negative events become rare, causing statistical uncertainty on E_c . For this reason, the parameters must be chosen such that $\langle \dot{w} \rangle$ is small, and fluctuation large. However, for short time-lags, as fluctuations are dominant, this asymmetry-based method becomes unreliable. The whole study relies on a delicate compromise.

The fluctuation rate $\sqrt{\left\langle \left(\Delta \dot{w}_{\tau}\right)^{2}\right\rangle /\left\langle \dot{w}_{\tau}\right\rangle}$ is decisive for the reliability of this measurement. In this study, it is between 20 and 60. It is a very useful feature of this experiment to adjust $\langle \dot{w}_{\tau} \rangle$ at will.

It can be noticed on fig. 3 that the histograms are close to Gaussian, when τ increases. This is expected from central limit theorem for variables such as \dot{w}_{τ} , resulting from a summation. This does not mean that \dot{w} is Gaussian, actually it is more like exponential, but the behavior at small τ are not of interest here. Besides, it has been checked that the variance goes as $1/\tau$ as the mean is constant. For a Gaussian variable, it can easily be proved that the FT

takes the very simple form:

$$E_c = \frac{\tau}{2} \frac{\langle \dot{w}_\tau^2 \rangle}{\langle \dot{w}_\tau \rangle}.$$
 (3)

Note that such a relation between the two sole moments of the Gaussian is remarkable: the FT relates it by introducing the characteristic energy.

In addition to the slope of the asymmetry function, the calculation of E_c has been performed thanks to the previous relation for Gaussian processes. The two independent methods are expected to give a close result.

Despite a large scattering of the slope measurements, fig. 7 shows that they agree within a range of about 20 to 30%. This discrepancy has been observed in all measurements. The process \dot{w}_{τ} is comparable to a Gaussian, but seems distinct: the preceding method is consistent, but cannot be used to measure E_c reliably, at this point.

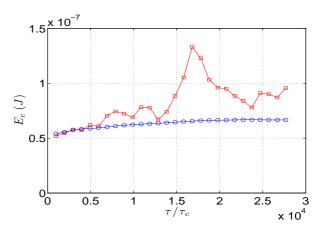


Fig. 7: E_c against τ/τ_c , τ_c being the correlation time of the process \dot{w} . The circles are values of E_c from the ratio of variance over mean, that gives E_c if the process is Gaussian. The squares gives E_c as the inverse slope of the asymmetry function. Acceleration: $a=56\,\mathrm{ms}^{-2}$.

It is interesting to compute the correlation time τ_c of \dot{w} . The loss of correlation is rather fast, as the level of the periodic excitation is not much larger than the noise of the gas. (See Fig. 4, top.) In all of the measurements shown, the correlation time τ_c is constantly of the order of the mS, which corresponds simply to the inverse of the spectrum frequency span. This microscopic time τ_c is possibly the mean flight time of the beads between two collisions.

The measurement described above is re-conducted for several values of the current I, at constant value of the vessel acceleration $a=41\,ms^{-2}$. The curves shown in fig. 8 collapse to a unique value $E_c\simeq 1.7\,10^{-7}\,\rm J$. Uncertainties are difficult to evaluate, but it is believed that the dispersion of the curves is mainly due to the fitting

process of the asymmetry function, and to statistical limitations

The collapse of the curves in fig. 8 obtained at different I, can be interpreted as the following. Within a certain range of current amplitudes, E_c does not depend on the blade-motor system and its excitation. It is therefore a characteristic of the granular gas solely.

In other words, this externally driven blade-motor system actually qualifies as a probe.

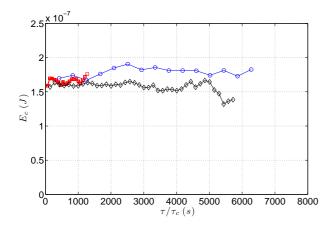


Fig. 8: E_c against τ , for several values of the excitation current, but the same acceleration: $a=41\,\mathrm{ms^{-2}}$. Squares, diamonds and circles refer respectively to currents $I=1\,\mathrm{mA}$, $700\mu\,\mathrm{A}$, and $400\mu\,\mathrm{A}$.

The range of acceptable currents is however bounded. If I is too large, angular velocity simply follow the torque and no negative events are observed: E_c cannot be measured. If I is too small, the histograms are almost even, causing a poor resolution on E_c . The range respecting this compromise is in practice large enough for measurement, but improvements of the protocol might certainly extend this range.

The following step is to relate this characteristic energy to some parameters of the NESS granular gas itself. Varying the power supply of the shaker $P_{\rm exc.}$, one easily vary the acceleration of the vessel and therefore of the granular gas. In fig. 9, E_c is plotted for several values of the acceleration against the variance of the induced voltage $\langle e^2 \rangle$ (e is proportional to the angular velocity $\dot{\theta}$). (These fluctuations of e has been recorded with no excitation.) These two quantities seems proportional, to a rather good approximation, suggesting that E_c could be interpreted as a kinetic energy of the blade, coupled to the beads in its vicinity.

Now, relatively to the acceleration. Supposing some sort of thermalisation between the blade and the gas, E_c can be written tentatively in the form: $E_c = E_0 + \langle mv^2 \rangle$. The

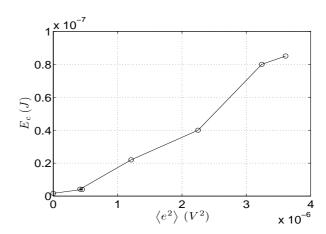


Fig. 9: E_c against $(a/(2\pi f_{\rm exc.}))^2$, for several values of the acceleration (between 41 ms⁻² and 59 ms⁻²). The frequency $f_{\rm exc.}=40$ Hz, and the current excitation of the motor is unchanged: $I=200~\mu{\rm A}$.

reduced mass m is accounting for a mass of beads, and v is their velocity. The brackets represent an average on an unspecified number of particles in the vicinity of the blade.

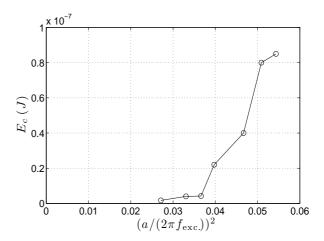


Fig. 10: E_c against $(a/(2\pi f_{\rm exc.}))^2$, for several values of the acceleration (between $41\,{\rm ms}^{-2}$ and $59\,{\rm ms}^{-2}$). The frequency $f_{\rm exc.}=40\,{\rm Hz}$, and the current excitation of the motor is unchanged: $I=200\,\mu{\rm A}$.

The threshold E_0 accounts for the energy necessary to excite the gas up to the height of the blade. Still dimensionally, this relation can be rewritten as: $E_c = E_0 + m'(a/(2\pi f_{\rm exc.}))^2$. Fig. 10 shows that the data are possibly described by a straight line in these coordinates, apart from the very small values. The results are in agreement with this naive picture, at least at the present level of uncertainty. Error bars on the measurement of E_c are

hard to quantify, but may be large. In these conditions it is vain to further investigate the parameters E_0 and m', which are system-dependent.

Conclusion and perspective. — The system presented here is very simple. Nevertheless, it allows very sensitive measurements and gives access to important and topical issues in non-equilibrium statistical mechanics.

It consists of a driven harmonic oscillator, coupled as a probe to a granular gas. It is composed by a DC-motor with a blade and a torsion spring on its axis. The granular gas is composed of a large number of beads, maintained in a gaseous state by an external power supply $P_{\rm exc.}$ from a shaker. The granular gas has an extremely large number of degrees of freedom, which guaranties an efficient chaotic loss of memory. It forms a NESS bath, to which the harmonic oscillator thermalises. The granular gas mimics quite well a real gas, yet with two important differences: the number of particles is not extremely large (fluctuations are important), the collisions are dissipative (a power supply is necessary).

The DC-motor driving the harmonic oscillator is used as actuator and sensor altogether, to probe the granular gas. The electro-mechanical relations in a DC-motor are such that no calibration is needed to measure the power delivered to the bath. This property is fortunate, as calibration is often a source of errors.

Through this device, a small energy flux \dot{w} is injected into the granular gas at controlled torque. Note that $\langle \dot{w} \rangle$ is of the order of a few 10^{-9} W, *i.e.* resolution is much less. It is extremely small in such a macroscopic experiment!

The fluctuations of this perturbation power is shown to verify the Stationary State Fluctuation Theorem. An output quantity, a characteristic energy E_c is measured, which does not depend on the imposed torque amplitude within a certain range. Therefore, one concludes that E_c is characteristic of the granular gas itself, *i.e.* this small system is actually a probing system.

For different values of the power $P_{\rm exc.}$ supplying the NESS, the characteristic energy E_c is measured, and found likely linked to the kinetic energy of the blade and beads in the vicinity. It is plotted against the acceleration. Despite important scattering of the points, a clear tendency to increase is visible, compatible with a quadratic dependance in $a/f_{\rm exc.}$. This corroborates the intuition.

A quantitative characterisation of this relation definitely requires improvements of the experimental set-up and the measurement procedure. The difficulty is to measure the asymptotic E_c . In other terms, the range of time coarse-graining τ to perform is the result of a delicate compromise. Time-lag τ must be large enough to overcome the transient and give an asymptotic value of the slope, but still allow enough negative fluctuations necessary to measure E_c at all! The constraints might be more difficult to satisfy altogether for a wide range of acceleration and excitation frequency.

Applying the FT to a small perturbative energy flux in-

stead of that supplying the NESS is a biased opinion adopted here, which appears fruitful. This approach allows in principle the measurement of negative fluctuations of injected power, with arbitrarily long statistics, for a granular gas as far as desired from equilibrium. This key point was discussed by Zamponi [7] as a limiting benchmark for experiments.

One interesting question is whether E_c obtained via the FT in the present study is the same or different as that obtained using the fluctuations of the power supply $P_{\text{exc.}}$, that maintains the gas in its NESS. (Like in most of previous studies, for instance in [3]).

It will be interesting in the future to understand the physical significance of E_c . The FT, as often considered, gives an interpretation of the characteristic energy in terms of an effective (non-equilibrium) temperature: $E_c \equiv k_{\rm B}T_{\rm eff}$. In the present experiment however, such interpretation deserves caution, as the forcing is non-Gaussian (for short times), the granular gas is intrinsically dissipative and has an irreversible dynamics. Interpretation of these measurements remains to be done.

* * *

I gratefully acknowledge S. Ciliberto, who has been encouraging and present as an expert adviser, all along this work as well as for other projects in progress. Many thanks to A. Steinberger and E. Bertin for their help. Many thanks to the colleagues and the students of the ÉNS-Lyon for so many discussions.

REFERENCES

- Evans D. J., and Cohen E. G. D., and Morris G. P., *Phys. Rev. Lett.*, **71** (1993) 2401, Gallavotti G., and Cohen E.D.G., *Phys. Rev. Lett.*, **74** (1995) 2694-2697
- [2] CILIBERTO S., and JOUBAUD S. and PETROSYAN A., J. Stat. Mech., 2010 (2010) P12003 and references therein.
- [3] Aumaître S. and Farago J. and Fauve S. and Mc Namara S., Eur. Phys. J. B, 42 (2004) 255-261
- [4] Puglisi A. and Visco P. and Barrat A. and Trizac E. and Van Wijland F., *Phys.Rev. Lett.*, **95** (2005) 110202, and references therein,
- [5] FEITOSA K. and MENON N., Phys. Rev. Lett., 92 (2004) 164301.
- [6] DANNA G. and MAYOR P. and BARRAT A. and LORETO V. and NORI F., *Nature*, **424** (2003) 909912.
- [7] ZAMPONI F., J. Stat. Mech., 2007 (P02008)