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Perturbative fluctuation dissipation relation for non-equilibrium finite frequency noise in quantum circuits

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We develop a general perturbative computation of finite-frequency quantum noise which applies, in particular, to both good or weakly transmitting strongly correlated conductors coupled to a generic environment. Under a minimal set of hypotheses, we show that the noise can be expressed through the non-equilibrium DC current only, generalizing a non-equilibrium fluctuation dissipation relation. We use this relation to derive explicit predictions for the non equilibrium finite frequency noise for a single channel conductor connected to an arbitrary Ohmic environment.

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In a quantum circuit composed of several coherent conductors, electronic transport depend on the global circuit even when conductors are separated by distances greater than the electronic coherence length as, for instance, in the dynamical Coulomb blockade (DCB)^{1,2}. Consequently, classical laws of electricity such as the impedance composition law are violated³. It is therefore important to look for quantum laws of electricity replacing the classical ones. They must be independent on details of the dynamics such as coupling to the environment or screened Coulomb interactions within each conductor. For example, the recently derived universal relation between current correlations and generalized admittances for non-linear time dependent transport in quantum circuits⁴ is a consistency condition valid independently of the details of the system's initial density matrix, Hamiltonian and coupling to its environment.

Assuming thermal equilibrium also leads to system independent relations such as the standard fluctuation dissipation theorem (FDT)^{5,6} derived within a linear response theory. This FDT is now embraced by a general corpus of non-equilibrium fluctuation relations derived for charge transport in the classical regime^{7,8}, non-linear DC transport through a single quantum conductor⁹ and quantum circuits in the limit of weak environmental effects¹⁰. However the validity of such results for quantum circuits involving strong environmental effects is still an open question.

Such effects have been studied within the Dynamical Coulomb Blockade problem, first in a tunnel junction coupled to a linear environment¹¹. The case of a good conductor coupled to a small impedance was then considered¹²⁻¹⁴ and further works¹⁵⁻¹⁷ have completed our understanding of the DCB of the current for higher impedances. The question of the DCB of the noise has been explored theoretically^{15,18,19}, leading to experimental investigations²⁰. The importance of fluctuation dissipation relations (FDRs) relating the finite frequency

(FF) quantum noise to the DC non-equilibrium average current progressively emerged^{21,22}. In fact, similar FDRs had been derived in the stationary regime in the fractional quantum Hall effect²³ and, for the symmetrized FF noise for free quasiparticles^{24,25}, in presence of a linear environment²⁶ and in presence of arbitrary interactions²⁷.

In this Letter, we show that the FDR between the FF quantum noise and DC non-equilibrium current²¹ is valid independently of the details of the conductor as well as of its environment provided the following hypotheses are satisfied : (i) validity and finiteness of perturbation theory, (ii) absence of superconducting current and (iii) detailed balance in the limit of vanishing tunneling, a condition to be obeyed by the total tunneling operator, which includes phase fluctuations.

This FDR can be exploited in various ways: first, it provides a test of the hypotheses (i)-(iii) when one measures independently both the average current and its FF noise. Secondly, it provides explicit predictions for the FF noise from the DC non-equilibrium current which is easier to compute than the noise. As an illustration, using the mapping between the DCB problem and the Tomonaga-Luttinger liquid (TLL) theory^{16,28}, our FDR gives explicit predictions for the effect of an Ohmic environment on the FF noise from that on the DC non-equilibrium current, thus extending previous^{19,20,23,29} works.

We consider a quantum circuit built from a two terminal conductor with possible strong correlations and coupled to an arbitrary environment involving other conductors without any restriction (see Figure 1).

The circuit is biased by a DC voltage V and its Hamiltonien is decomposed into

$$\mathcal{H}(t) = \mathcal{H}_0 + e^{i\omega_{\text{dct}} t} \mathcal{A} + e^{-i\omega_{\text{dct}} t} \mathcal{A}^\dagger. \quad (1)$$

Here \mathcal{H}_0 involves internal or mutual Coulomb interactions as well as the environment. Charge transfer pro-

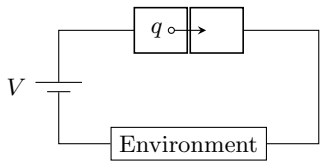


FIG. 1: A general two terminal conductor is embedded into a quantum circuit in which it is coupled to an impedance $Z(\omega)$ and/or other conductors constituting its environment. Here the conductor is a spatially extended tunnel junction with capacitive couplings.

cesses across the junction are encoded within tunneling operators \mathcal{A} and \mathcal{A}^\dagger which include environment induced quantum fluctuations of the phase jump across the conductor. The DC bias V introduces a frequency scale $\omega_{\text{dc}} = qV/\hbar$ where q denotes the effective renormalized charge of transferred quasi-particles. The current operator is defined as:

$$\hat{I}(t) = \frac{iq}{\hbar} (e^{i\omega_{\text{dc}}t} \mathcal{A} - e^{-i\omega_{\text{dc}}t} \mathcal{A}^\dagger). \quad (2)$$

Here we focus on FF noise expressed perturbatively with respect to \mathcal{A} . Several approaches to this problem are possible. One could model the electromagnetic environment in a quantum input/output approach, a first step in this direction being taken by Parlavacchio *et al* for a tunnel junction coupled to an LC oscillator²². On the other hand, the standard field theoretical approach followed here considers an adiabatic branching of tunneling from an initial condition at $t \rightarrow -\infty$ described by a density operator $\hat{\rho}_0$. Since $\hat{I}(t)$ is of first order with respect to \mathcal{A} , averages can be computed with respect to $\hat{\rho}_0$ and using the Heisenberg representation with respect to \mathcal{H}_0 :

$$\mathcal{A}_0(t) = e^{i\mathcal{H}_0 t} \mathcal{A} e^{-i\mathcal{H}_0 t} \quad (3)$$

To ensure absence of a super current in the limit of vanishing tunneling whenever superconductors are present, we require that:

$$\langle \mathcal{A}_0(t) \mathcal{A}_0(0) \rangle_0 = 0, \quad (4)$$

where $\langle \dots \rangle_0 = \text{tr}[\hat{\rho}_0 \dots]$. At the lowest order, the FF noise is given by

$$S(\omega, \omega_{\text{dc}}) = \int e^{i\omega(t-t')} \langle \hat{I}_0(t') \hat{I}_0(t) \rangle_0 dt' \quad (5)$$

where $\hat{I}_0(t)$ is obtained by replacing \mathcal{A} in Eq. (2) by $\mathcal{A}_0(t)$ given by Eq. (3). The detailed balance (hypothesis (iii)) constrains the occupation probabilities of the many body eigenstates of the circuit in the limit of vanishing tunneling to be given by a thermal distribution ρ_0 with effective temperature T . This enables us to relate the FF quantum noise to the DC non-equilibrium current across the circuit (see Appendix A):

$$S(\omega, \omega_{\text{dc}}) = q [N(\omega_{\text{dc}} + \omega) I(\omega_{\text{dc}} + \omega) + (1 + N(\omega_{\text{dc}} - \omega)) I(\omega_{\text{dc}} - \omega)] \quad (6)$$

where $I(\omega_{\text{dc}})$ denotes the non equilibrium dc current at bias voltage V and $N(\omega)$ denotes the Bose occupation number at temperature T . This perturbative non-equilibrium FDR is the central result of this Letter. It is model independent and its validity solely relies on hypotheses (i)-(iii). Note that Eq. (6) also implies the FDR for the symmetrized noise previously derived in more specific contexts^{24,26,27}.

Let us now analyze the various regimes of the FF noise $S(\omega, \omega_{\text{dc}})$ as a function of ω and ω_{dc} at fixed temperature T . First of all, its asymmetry with respect to ω is related to the non-equilibrium admittance $G(\omega, \omega_{\text{dc}})$ ⁴. As a consequence, both quantities have a perturbative expression in terms of the non-equilibrium DC current³⁰:

$$S(-\omega, \omega_{\text{dc}}) - S(\omega, \omega_{\text{dc}}) = 2\hbar\omega \Re(G(\omega, \omega_{\text{dc}})) \quad (7a)$$

$$= q \{I(\omega_{\text{dc}} + \omega) - I(\omega_{\text{dc}} - \omega)\}. \quad (7b)$$

Secondly, the noise $S(\omega, \omega_{\text{dc}})$ is even with respect to ω_{dc} whenever particle-hole symmetry holds: in that case, a spectral decomposition shows that the DC characteristic is odd. Generically $I(\omega_{\text{dc}})$ has the same sign as ω_{dc} and $I(\omega_{\text{dc}} = 0) = 0$.

Being mostly interested in the quantum regime, we shall explore frequencies ω_{dc} and ω well above the thermal scale $k_B T/\hbar$. Since $\hbar\omega$ represents the energy of photons emitted ($\omega > 0$) or absorbed ($\omega < 0$) by the conductor, it is natural to compare it to the energy scale $|qV|$. This leads us to partition the $(\omega, \omega_{\text{dc}})$ plane into four quadrants (see Fig. 2) separated by diagonal bands $\hbar|\omega \pm \omega_{\text{dc}}| \lesssim k_B T$ in which thermal fluctuations turn out to play a role even in the quantum regime.

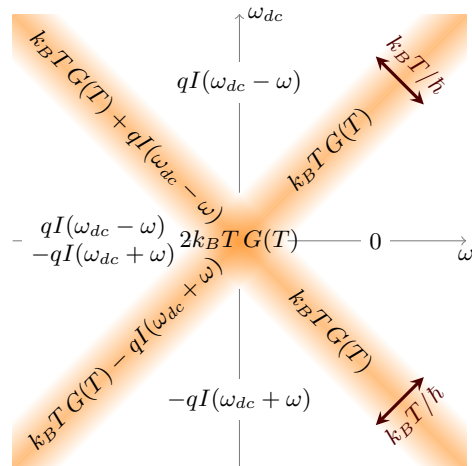


FIG. 2: (Color online) Various regimes for the noise as a function of ω and ω_{dc} . The $(\omega, \omega_{\text{dc}})$ plane is partitioned into four quadrants separated by diagonal bands of width $k_B T/\hbar$ (orange) corresponding to the typical energy scale of thermal electron/hole pairs. The quantum regime corresponds to $\hbar|\omega| \gg k_B T$ or $\hbar|\omega_{\text{dc}}| \gg k_B T$ whereas the thermal regime corresponds to both $\hbar\omega$ and $\hbar\omega_{\text{dc}}$ much smaller than $k_B T$.

We first look at the physics far from these bands. In the $\omega > |\omega_{\text{dc}}|$ quadrant (see Fig. 2), the system cannot emit any photon (at first order in perturbation theory). Therefore, we expect the FF noise to vanish⁴: $S(\omega, \omega_{\text{dc}}) = 0$. Then, Eq. (7) leads to the expression of the FF noise in the $\omega < -|\omega_{\text{dc}}|$ quadrant where it represents the ability of the circuit to absorb radiation. It is then naturally related to the dissipative part of the non equilibrium admittance (see Eq. (7)): $S(\omega, \omega_{\text{dc}}) = 2\hbar\omega\Re(G(\omega, \omega_{\text{dc}}))$. When particle-hole symmetry holds, $S(-\omega, 0) = 2qI(\omega)$ which reduces to the usual expression for the FF equilibrium noise only for a linear system.

We now consider the off-diagonal quadrants $|\omega_{\text{dc}}| > |\omega|$: the vanishing of Bose occupation numbers there implies that the FF noise is proportional $I(\omega_{\text{dc}} - \omega)$ (see Fig. 2). In particular, at zero frequency and for $|qV| \gg k_B T$, the noise has a Poissonian expression:

$$S(0, \omega_{\text{dc}}) = qI(\omega_{\text{dc}}) \quad (8)$$

shown here to be valid for an extended and interacting tunneling region^{31,32} with a generic environment whereas it was originally derived for an isolated tunnel junction between decoupled conductors³³.

Let us then look into diagonal bands where thermal fluctuations generate neutral excitations with energies below $k_B T$. In the thermal regime, when both ω and ω_{dc} are smaller than $k_B T/\hbar$, we recover the Johnson Nyquist³⁴ noise $S(0, 0) = 2k_B T G(T)$ where $G(T) = G(\omega = 0, \omega_{\text{dc}} = 0, T)$ is the linear conductance which may depend on temperature. Remarkably, at positive frequencies, in the quantum regime, the FF noise along the diagonals is half of the equilibrium noise whereas for negative frequencies, it picks up an extra non-equilibrium contribution $\pm qI(\omega_{\text{dc}} - \omega)$.

At finite frequency, one usually measures the differential of the FF noise with respect to the DC voltage or equivalently ω_{dc} . In particular, at low frequency, Eq. (6) implies that this differential noise is related to the differential conductance for $|qV| \ll k_B T$:

$$\left(\frac{\partial S}{\partial \omega_{\text{dc}}}\right)_{\omega=\omega_{\text{dc}}=0}(T) = k_B T \left(\frac{\partial G}{\partial \omega_{\text{dc}}}\right)_{\omega=\omega_{\text{dc}}=0}(T). \quad (9)$$

This relation has been derived for weak environmental effects¹⁰ and for an isolated interacting quantum conductor within a Hartree framework⁹. It has been experimentally tested in quantum dots³⁵.

To illustrate our general result, we will now use Eq. (6) to obtain explicit predictions for the FF noise of a single channel conductor in series with an Ohmic impedance R . As shown theoretically¹⁵ and confirmed experimentally¹⁶, this situation can be described in terms of a localized barrier in a Luttinger liquid (TLL)^{36,37} with interaction parameter $K = (1 + R/R_q)^{-1} < 1$ where $R_q = h/e^2$. A stronger coupling to the environment thus corresponds to stronger repulsive interactions in the TLL. The tunneling regime ($R_q G \ll 1$) of the DCB problem corresponds to a strong barrier in the TLL, *i.e.* to the vicinity

of its IR fixed point ($K < 1$) whereas a good conductor ($R_q G \simeq 1$) corresponds to the case of a weak barrier, *i.e.* to the vicinity of its unstable UV fixed point³⁷.

Consequently, for a good conductor, we expect perturbative results to be valid when the largest energy among $|qV|$ and $k_B T$ is greater than E_B , an intrinsic energy of this impurity problem scaling as $E_B \simeq \hbar\omega_{\text{RC}}(1 - \mathcal{T})^{1/2(1-K)}$ in terms of $\mathcal{T} = R_q G(R/R_q = 0) \lesssim 1$, the bare transmission (no DCB) of the conductor^{15,16} and of the UV cutoff of the problem ω_{RC} . A linear DC characteristic $V = (R + R_q)I$ corresponding to the series addition of resistances is recovered when $k_B T \gg |qV|$ and $\max(k_B T, |qV|) \gg E_B$. For a good conductor, voltage division within the circuit then leads to a charge renormalization $q = -eR_q/(R + R_q) = -eK$. When $|qV| \gg k_B T$, keeping $|qV| \gg E_B$, the DC characteristic is no longer linear: $I(V, T) = KV/R_q - I_B(V, T)$ where $I_B(V, T)$ is the weak backscattering current. Predictions for the backscattering noise $S_B(\omega, \omega_{\text{dc}})$ follow from Eq. (6) and from the perturbative expression³⁸:

$$I_B(V, T) = V G_{\Delta}(T) \frac{\sinh\left(\frac{qV}{2k_B T}\right) \left|\Gamma\left(\Delta + \frac{iqV}{2\pi k_B T}\right)\right|^2}{\frac{qV}{2k_B T} \Gamma(\Delta)^2} \quad (10)$$

where Δ is equal to $K < 1$, $q = -Ke$ and $G_{\Delta}(T)$ is a backscattering conductance scaling as $G_{\Delta}(T)/G_{\Delta=1}(T) = (\Gamma(\Delta)^2/\Gamma(2\Delta)) \times (\hbar\omega_{\text{RC}}/\pi k_B T)^{2(1-\Delta)}$ (see Appendix B). In fact, Eq. (10) is valid as long as $\max(|qV|, k_B T) > E_B$. Consequently, for $k_B T \gg E_B$, Eq. (10) can be used in Eq. (6) without further restrictions on $(\omega, \omega_{\text{dc}})$ whereas, at lower temperatures ($k_B T \lesssim E_B$), our result for $S_B(\omega, \omega_{\text{dc}})$ is valid for $\hbar|\omega_{\text{dc}} \pm \omega|$ greater than E_B .

Remarkably, the mapping of the DCB on the TLL model also gives access to the low energy behavior ($|qV| \ll E_B$ and $k_B T \ll E_B$) of a good conductor. This behavior also describes the low energy physics of a tunnel junction due to the strong DCB¹⁵. Perturbation theory can then be applied close to the IR fixed point corresponding to a disconnected TLL. Since the total effective resistance of the circuit is now much larger than R , no voltage division takes place thus giving $q = -e$. Eq. (10) can then be used to compute $I(V, T)$ with $\Delta = K^{-1} = 1 + R/R_q > 1$ and $G_{\Delta}(T)$ then corresponds to the linear conductance $G(T)$. Predictions for $S(\omega, \omega_{\text{dc}})$ from Eq. (6) are now valid when $|\hbar\omega| \ll E_B$ and $|qV| \ll E_B$. They apply as well to a weakly transmitting conductor where perturbation theory is expected to be valid for all energy scales smaller than the cutoff energy $\hbar\omega_{\text{RC}}$.

To understand the effect of the environment on the FF noise in both regimes, we plot the ratio $S_R(\omega, \omega_{\text{dc}})$ of the noise for $R \neq 0$ to its value at $R = 0$ (no DCB). We present our results assuming $\hbar\omega_{\text{RC}} = 40E_B$ and for two values of R : $R = R_q/2$ ($K = 2/3$) and $R = 2R_q$ ($K = 1/3$). Note that the FF noise is even with respect

to ω_{dc} due to the electron/hole symmetry of the TLL model.

Figure 3 presents $\mathcal{S}_R(\omega, \omega_{\text{dc}})$ for a good conductor, assuming $k_B T/E_B = 5$. In this regime, $\mathcal{S}_R(\omega, \omega_{\text{dc}}) \geq 1$: the environment enhances the backscattering noise. This enhancement is especially strong in the thermal regime and for the emission noise within the diagonal bands $|\omega - |\omega_{\text{dc}}|| \lesssim k_B T/\hbar$. It becomes even stronger and more concentrated along the diagonal bands with increasing R . This apparently surprising result comes from the fact that increasing R leads to a stronger DCB of the total current corresponding to an increase of I_B and correspondingly of its FF noise.

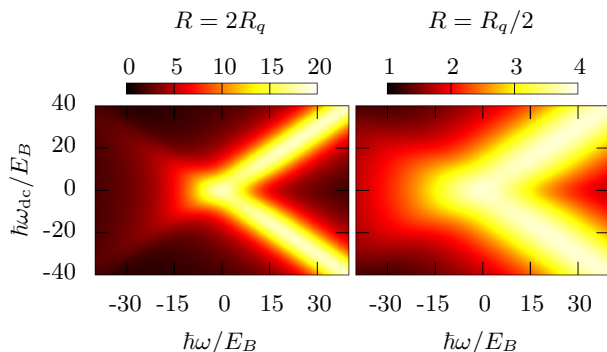


FIG. 3: (Color online) Case of a good conductor: Density plots of the ratio $\mathcal{S}_R(\omega, \omega_{\text{dc}})$ of the backscattering current noise for $R \neq 0$ to the same quantity at $R = 0$ (no environment) as function of $\hbar\omega/E_B$ and $\hbar\omega_{\text{dc}}/E_B$ for $R = 2R_q$ ($K = 1/3$) and $R = R_q/2$ ($K = 2/3$) assuming $k_B T = 5 E_B$. Frequencies ω and ω_{dc} are kept below the high energy cutoff $\hbar\omega_{\text{RC}} = 40 E_B$.

Figure 4 presents $\mathcal{S}_R(\omega, \omega_{\text{dc}})$ in the strong backscattering regime describing both a weakly transmitting conductor and the low energy behavior of a good conductor. At low temperature $k_B T = E_B/10$, a DCB of the noise is observed as expected from the DCB of the DC non-equilibrium current.

Based on a generalized mapping between the DCB problem and a generalized TLL model^{16,29}, we expect these conclusions on noise enhancement/reduction by environmental effect to remain qualitatively valid for a linear environment with a frequency dependent impedance.

To conclude, we have obtained a non-equilibrium perturbative FDR relating the FF noise to the non-equilibrium DC current across a generic two-terminal quantum circuit. This model independent FDR unifies many previous results and only relies on three hypotheses among which a detailed balance condition in the limit of vanishing tunneling. Most importantly, this condition is deeply related to the effective thermalization of the whole circuit, a question of first importance in mesoscopic thermodynamics³⁹. This out of equilibrium FDR opens the way to numerous experiments: first of all, test-

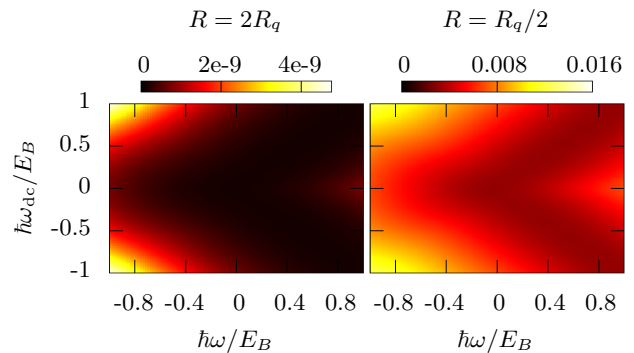


FIG. 4: (Color online) Weakly transmitting regime: Density plot of the ratio $\mathcal{S}_R(\omega, \omega_{\text{dc}})$ of the transmitted current noise for $R \neq 0$ to the same quantity at $R = 0$ (no environment) as function of $\hbar\omega/E_B$ and $\hbar\omega_{\text{dc}}/E_B$ for $R = 2R_q$ ($K = 1/3$) and $R = R_q/2$ ($K = 2/3$) for $\hbar\omega_{\text{RC}} = 40 E_B$ and $k_B T = E_B/10$. The plot is limited to energy scales below E_B to explore the vicinity of the IR fixed point of the TLL model.

ing it on complex nano-structures such as quantum dots would check the basic hypotheses (i)–(iii). Our FDR can also be used to determine whether one measures the symmetrized or non-symmetrized noise when accessing only the emission part of the noise spectrum ($\omega > 0$)²⁰. Finally, our non-equilibrium FDR and its generalization to AC bias²¹ provide complementary methods to measure the effective tunneling charge q in addition to these proposed in Ref.³⁰.

Acknowledgments

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Appendix A: Derivation of the Fluctuation Dissipation Relation

The first step is to compute, to the lowest relevant order in perturbation theory, both the average current $\langle I(t) \rangle$ and current correlations defined in the time domain by

$$S_I(t, t') = \langle I(t) I(t') \rangle - \langle I(t) \rangle \langle I(t') \rangle \quad (\text{A1})$$

Eq. (A1) defines a quantum (non-symmetrized) correlation function which, in full generality, is not symmetric with respect to $t \leftrightarrow t'$. The non-symmetrized current

noise at finite frequency is then defined by

$$S_I(\omega) = \int_{-\infty}^{+\infty} e^{i\omega\tau} \overline{S_I\left(\bar{t} - \frac{\tau}{2}, \bar{t} + \frac{\tau}{2}\right)} d\tau \quad (\text{A2})$$

where $\overline{S_I\left(\bar{t} - \tau/2, \bar{t} + \tau/2\right)}$ denotes the time average over $\bar{t} = (t+t')/2$ of $S_I(t, t')$ arising from the long acquisition time of the noise signal. With this definition, the emission noise corresponds to $\omega > 0$ whereas the absorption noise corresponds to $\omega < 0$. Generic current noise measurement corresponds to emission noise measurements⁴⁰ but it is now possible to access the full non-symmetrized noise by exploiting the photo-assisted tunneling of quasi-particles across an on-chip superconductor-insulator-superconductor junction⁴¹⁻⁴⁴. In the recently performed experiments on the dynamical Coulomb blockade of the noise^{20,22}, the detection setup accesses the non-symmetrized excess noise due to the AC bias of the quantum circuit even if the final detection stage is a standard power measurement.

1. Perturbation theory

Results are most easily obtained by going into the iteration representation with respect to the Hamiltonian \mathcal{H}_0 (no tunneling) and expand the evolution operators in powers of the operators $\mathcal{A}_0(t')$ and $\mathcal{A}_0^\dagger(t')$ which include environmental phases, expressed in the interaction scheme with respect to the Hamiltonian \mathcal{H}_0 .

a. The average current The zero-th order term trivially vanishes and, at the lowest non-trivial order, half of the terms involve either two operators \mathcal{A} or two operators \mathcal{A}^\dagger . These terms are assumed to vanish due to hypothesis (ii). The remaining terms involve exactly one \mathcal{A} and one \mathcal{A}^\dagger operator. They can therefore be expressed in terms of the two correlators:

$$X^>(t, t') = \text{Tr}(\mathcal{A}_0(t) \rho_0 \mathcal{A}_0^\dagger(t')) \quad (\text{A3a})$$

$$X^<(t, t') = \text{Tr}(\mathcal{A}_0^\dagger(t) \rho_0 \mathcal{A}_0(t')). \quad (\text{A3b})$$

Consequently, at lowest order, under our hypotheses, the average current at time t is obtained as

$$\langle I(t) \rangle = \frac{q}{\hbar^2} \int_0^t E(t') E(t)^* (X^<(t', t) - X^>(t, t')) dt' \quad (\text{A4a})$$

$$+ \frac{q}{\hbar^2} \int_0^t E(t')^* E(t) (X^>(t, t') - X^<(t, t')) dt'. \quad (\text{A4b})$$

b. Current noise Let us now turn to the current noise. In this case, the second order in perturbation theory is obtained without expanding the evolution operators since we have already products of two operators \mathcal{A} and \mathcal{A}^\dagger . Since the product $\langle I(t) \rangle \langle I(t') \rangle$ is at fourth order,

the current noise is finally given by:

$$S_I(t, t') = \frac{q^2}{\hbar} (E(t)^* E(t') X^>(t', t) + E(t')^* E(t) X^<(t', t)). \quad (\text{A5})$$

Although these relations are valid for a general time dependent voltage and are thus relevant for discussing photo-assisted noise and current, we shall now focus on the stationary regime where $V(t) = V$ and therefore $E(t) \propto e^{-i\omega_{\text{dc}}t}$. As we shall see now, the above expressions will simplify and provide an explicit FDR relating the noise spectrum of the quantum noise to the non-equilibrium dc characteristic of the conductor.

2. Stationary case

In the stationary case, the tunneling correlators $X^>(t, t')$ and $X^<(t, t')$ only depend on the difference $\tau = t - t'$. Introducing the Fourier transforms

$$X^>(\omega) = \int_{-\infty}^{+\infty} X^>\left(\frac{\tau}{2}, -\frac{\tau}{2}\right) e^{i\omega\tau} d\tau \quad (\text{A6a})$$

$$X^<(\omega) = \int_{-\infty}^{+\infty} X^<\left(\frac{\tau}{2}, -\frac{\tau}{2}\right) e^{i\omega\tau} d\tau, \quad (\text{A6b})$$

and using Eqs. (A4), the non-equilibrium DC current is obtained as ($\omega_{\text{dc}} = qV/\hbar$):

$$I(\omega_{\text{dc}}) = \frac{q}{\hbar^2} (X^<(-\omega_{\text{dc}}) - X^>(\omega_{\text{dc}})). \quad (\text{A7})$$

Substituting Eq. (A2) into Eq. (A5) then leads to the FF noise:

$$S_I(\omega) = \frac{q^2}{\hbar^2} (X^<(\omega - \omega_{\text{dc}}) + X^>(\omega + \omega_{\text{dc}})). \quad (\text{A8})$$

The detailed balance (hypothesis (iii)) relates the occupation probabilities p_I of the many body eigenstates $|I\rangle$ of the circuit with energies E_I in the limit of vanishing tunneling:

$$\frac{p_I}{p_J} = e^{-(E_I - E_J)/k_B T}. \quad (\text{A9})$$

Using (A9) within the Källén-Lehmann spectral representation of $X^>(t, t')$ and $X^<(t, t')$, the Fourier transforms $X^>(\omega)$ and $X^<(-\omega)$ can be related through

$$X^<(-\omega) = e^{\hbar\omega/k_B T} X^>(\omega). \quad (\text{A10})$$

This enables us to express $X^>(\omega)$ as well as $X^<(-\omega)$ in terms of the dc out of equilibrium current:

$$q X^>(\omega) = \hbar^2 N(\omega) I(\omega) \quad (\text{A11a})$$

$$q X^<(-\omega) = \hbar^2 (N(\omega) + 1) I(\omega) \quad (\text{A11b})$$

where $N(\omega) = (e^{\hbar\omega/k_B T} - 1)^{-1}$ denotes the Bose occupation number and, in the above expressions, this expression is also used to define $N(\omega)$ $\omega < 0$. Substituting

Eq. (A11) into Eq. (A8) leads to our main result, *i.e.* the expression of the FF noise in terms of the out of equilibrium dc current:

$$S_I(\omega) = q(N(\omega_{\text{dc}} + \omega)I(\omega_{\text{dc}} + \omega) + (1 + N(\omega_{\text{dc}} - \omega))I(\omega_{\text{dc}} - \omega)). \quad (\text{A12})$$

Appendix B: The Tomonaga Luttinger barrier problem

1. Presentation of the problem

The problem of a single localized barrier in the TLL has been originally studied by Kane and Fisher within a renormalization group approach³⁶. In the interacting case ($K \neq 1$), their work has revealed a phase diagram showing an UV and an IR fixed point respectively corresponding to a fully transmitting conductor and a disconnected conductor. These fixed points exchange their stability between the repulsive case $K < 1$ and the attractive case $K > 1$. In the repulsive case, the UV fixed point is unstable whereas the IR fixed point is stable, thus implying that the effective barrier diverges at low energies whereas it vanishes at low energies.

A full solution of the problem has been provided in the non-equilibrium stationary case using the thermodynamical Bethe ansatz technique, thus allowing a full interpolation between these two fixed points and explicit predictions for the non-equilibrium current and the low frequency noise^{45,46} with natural applications to the fractional quantum Hall effect (FQHE)⁴⁷. Further work has led to the determination of the full counting statistics of the charge flowing across a quantum point contact in the FQHE⁴⁸ and in the TLL⁴⁹. More recently, an exact description of non-equilibrium fixed points of quantum impurity models suitable for treating time-dependant problems has been proposed⁵⁰ and may open a non-perturbative approach to generalize or go beyond the perturbative results discussed here.

In the present letter, we shall only use perturbative results for the out of equilibrium current in the vicinity of both fixed points. Such explicit perturbative expressions have been obtained for the non-equilibrium current in the vicinity of the UV fixed point by Chamon *et al*³⁸ but the duality of the local barrier problem in a TLL^{51,52} enables us to use similar expressions close to the IR fixed point provided one replaces K by $1/K$.

2. The weak backscattering regime

In this regime, the barrier is modeled by a localized potential described by a localized potential v_B . The model also has a high frequency cutoff denoted here by ω_{RC} . A relation between microscopic parameters v_B and ω_{RC} and measurable quantities is obtained by considering $K = 1$

which corresponds to the bare conductor ($R = 0$ in the DCB problem). Then denoting $\mathcal{T} = G_{K=1}(T)$ which indeed does not depend anymore on the temperature, we find:

$$\mathcal{T} = 1 - \frac{(\pi v_B)^2}{\omega_{\text{RC}}} \quad (\text{B1})$$

Let us now consider $K < 1$ directly relevant for the weak backscattering regime since $K = (1 + R/R_q)^{-115}$.

At first non-trivial order in perturbation theory, the total current flowing across the barrier contains a weak backscattering correction to the bare current $q^2 KV/h$:

$$I(V, T) = \frac{e^2}{h} V \left(K - G_K(T) \mathcal{F}_K \left(\frac{qV}{k_B T} \right) \right) \quad (\text{B2})$$

where

$$\mathcal{F}_K(z) = \frac{\sinh(z/2)}{z/2} \frac{|\Gamma(K + \frac{iz}{2\pi})|^2}{\Gamma(K)^2} \quad (\text{B3})$$

and $G_K(T)$ is a dimensionless backscattering conductance whose expression depends on the cutoff as well as on the potential of the barrier:

$$G_K(T) = \pi^2 \frac{\Gamma(K)^2}{\Gamma(2K)} \frac{v_B^2}{\omega_{\text{RC}}} \left(\frac{\hbar \omega_{\text{RC}}}{\pi k_B T} \right)^{2(1-K)} \quad (\text{B4})$$

Using Eq. (B1), we can then reexpress the dimensionless backscattering linear conductance $G_K(T)$ in presence of interactions in terms of the temperature and of the energy scale E_B associated with the barrier:

$$G_K(T) = \frac{\Gamma(K)^2}{\Gamma(2K)} \left(\frac{E_B}{\pi k_B T} \right)^{2(1-K)} \quad (\text{B5})$$

where E_B is related to the microscopic parameters through the relation

$$E_B = \hbar \omega_{\text{RC}} (1 - \mathcal{T})^{1/2(1-K)}. \quad (\text{B6})$$

valid in the limit $\mathcal{T} \sim 1$.

3. The strong backscattering regime

In the strong backscattering regime, the system is modeled by two half infinite 1D TLLs coupled by a tunneling barrier described by a tunneling amplitude Γ . At the IR fixed point ($\Gamma = 0$), no current flows across the barrier. When switching on tunneling, current can flow but, at zero temperature, interactions lead to a non-linear non-equilibrium current in terms of the dc bias³⁶. At non zero temperature, a temperature dependent linear conductance can still be defined and vanishes with temperature/ Nevertheless, second order perturbation theory in Γ leads to the following expression for the non-equilibrium current:

$$I(V, T) = \frac{e^2}{h} V G_{1/K}(T) \mathcal{F}_{1/K} \left(\frac{qV}{k_B T} \right) \quad (\text{B7})$$

where $\mathcal{F}_{1/K}(z)$ is given by Eq. (B3) replacing K by $1/K$ and $G_{1/K}(T)$ denotes the linear conductance at finite temperature T whose expression is given by substituting K by $1/K$ and v_B by Γ in Eq. (B5). As expected, $G_{1/K}(T)$ vanishes at low temperature for $K < 1$.

4. Limitations to the perturbative approach for a good conductor

Here we discuss how the breakdown of perturbation theory manifests itself in predictions of the FF noise deduced from Eq. (A12). Let us consider here the case of a good conductor, keeping $|\hbar\omega|$ and $|qV|$ smaller than the high energy cutoff $\hbar\omega_{RC}$.

In order to see the breakdown of perturbation theory when one leaves the vicinity of the UV fixed point of the TLL barrier problem, let us lower the temperature, starting from $k_B T \gtrsim E_B$. Fig. 5 depicts the prediction

for the dimensionless ratio

$$\mathcal{S}_R(\omega, \omega_{dc}) = \frac{S_{R,T}(\omega, \omega_{dc})}{S_{R=0,T}(\omega, \omega_{dc})} \quad (\text{B8})$$

associated with the backscattering current for three values of the temperature: $k_B T/E_B = 2, 0.5$ and 0.1 .

Fig. 5 clearly shows the signs of a divergence along in diagonal bands $|\omega - |\omega_{dc}|| \lesssim E_B/\hbar$ when decreasing the temperature below E_B/k_B . This is a signature of the breakdown of perturbation theory since, in this region, when $k_B T$ becomes smaller than E_B , all energy scales involved in the non-equilibrium current become smaller than E_B . This is precisely the regime where perturbation theory close to the UV fixed point is expected to break down in the TLL barrier problem. Infrared divergences appearing along the diagonal band expresses the divergence of the low energy fluctuations associated with neutral excitations (e/h pairs).

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- ¹ M.H. Devoret, D. Esteve, H. Grabert, G.-L. Ingold, H. Pothier, and C. Urbina, Phys. Rev. Lett. **64**, 1824 (1990).
 - ² S.M. Girvin, L.I. Glazman, M. Jonson, D.R. Penn, and M.D. Stiles, Phys. Rev. Lett. **64**, 3183 (1990).
 - ³ C. Altimiras, U. Gennser, A. Cavanna, D. Mailly, and F. Pierre, Phys. Rev. Lett. **99**, 256805 (2007).
 - ⁴ I. Safi and P. Joyez, Phys. Rev. B **84**, 205129 (2011).
 - ⁵ H. Callen and T. Welton, Physical Review **83**, 34 (1951).
 - ⁶ R. Kubo, J. Phys. Soc. Jpn. **12**, 570 (1957).
 - ⁷ A. Altland, A. De Martino, R. Egger, and B. Narozhny, Phys. Rev. B **82**, 115323 (2010).
 - ⁸ A. Altland, A. De Martino, R. Egger, and B. Narozhny, Phys. Rev. Lett. **105**, 170601 (2010).
 - ⁹ H. Förster and M. Büttiker, Phys. Rev. Lett. **101**, 136805 (2008).
 - ¹⁰ J. Tobiska and Y. V. Nazarov, Phys. Rev. B **72**, 235328 (2005).
 - ¹¹ G.-L. Ingold and Y. Nazarov, *Single charge tunneling* (Plenum Press, New York, 1992), vol. 294 of *NATO ASI Series B*, chap. Charge tunneling rates in ultrasmall junctions, pp. 21–107.
 - ¹² D.S. Golubev and A.D. Zaikin, Phys. Rev. Lett. **86**, 4887 (2001).
 - ¹³ A. A. Lévy Yeyati, A. Martin-Rodero, D. Estève, and C. Urbina, Phys. Rev. Lett. **87**, 046802 (2001).
 - ¹⁴ M. Kindermann and Y.V. Nazarov, Phys. Rev. Lett. **91**, 136802 (2003).
 - ¹⁵ I. Safi and H. Saleur, Phys. Rev. Lett. **93**, 126602 (2004).
 - ¹⁶ S. Jezouin, M. Albert, F. Parmentier, A. Anthore, U. Gennser, A. Cavanna, I. Safi, and F. Pierre, Nature Communications **4**, 1802 (2013).
 - ¹⁷ D. S. Golubev, A. V. Galaktionov, and A. D. Zaikin, Phys. Rev. B **72**, 205417 (2005).
 - ¹⁸ M. Kindermann, Y. V. Nazarov, and C. W. J. Beenakker, Phys. Rev. B **69**, 035336 (2004).
 - ¹⁹ R. Zamoum, A. Crépieux, and I. Safi, Phys. Rev. B **85**, 125421 (2012).
 - ²⁰ C. Altimiras, O. Parlavecchio, P. Joyez, D. Vion, P. Roche, D. Esteve, and F. Portier, Phys. Rev. Lett. **112**, 236803 (2014).
 - ²¹ I. Safi (2014), Preprint arXiv:1401.5950.
 - ²² O. Parlavecchio, C. Altimiras, J.-R. Souquet, P. Simon, I. Safi, P. Joyez, D. Vion, P. Roche, D. Estève, and F. Portier, Phys. Rev. Lett. **114**, 126801 (2015).
 - ²³ C. Bena and I. Safi, Phys. Rev. B **76**, 125317 (2007).
 - ²⁴ D. Rogovin and D. Scalapino, Annals of Physics **86**, 1 (1974).
 - ²⁵ A.J. Dahm, A. Dennenstein, D.N. Langenberg, W.H. Parker, D. Rogovin and D.J. Scalapino, Phys. Rev. Lett. **22**, 1416 (1969).
 - ²⁶ H. Lee and L.S. Levitov, Phys. Rev. B **53**, 7383 (1996).
 - ²⁷ E.V. Sukhorukov, G. Burkard, and D. Loss, Phys. Rev. B **63**, 125315 (2001).
 - ²⁸ I. Safi, P. Devillard, and T. Martin, Phys. Rev. Lett. **86**, 4628 (2001).
 - ²⁹ J.-R. Souquet, I. Safi, and P. Simon, Phys. Rev. B **88**, 205419 (2013).
 - ³⁰ I. Safi and E. V. Sukhorukov, EPL (Europhysics Letters) **91**, 67008 (2010).
 - ³¹ I. Safi (1999), unpublished.
 - ³² D. Chevallier, J. Rech, T. Jonckheere, C. Wahl, and T. Martin, Phys. Rev. B **82**, 155318 (2010).
 - ³³ L. S. Levitov and M. Reznikov, Phys. Rev. B **70**, 115305 (2004).
 - ³⁴ H. Nyquist, Physical Review **32**, 110 (1928).
 - ³⁵ S. Nakamura, Y. Yamauchi, M. Hashisaka, K. Chida, K. Kobayashi, T. Ono, R. Leturcq, K. Ensslin, K. Saito, Y. Utsumi, et al., Phys. Rev. Lett. **104**, 080602 (2010).
 - ³⁶ C.L. Kane and M.P.A. Fisher, Phys. Rev. B **46**, 15233 (1992).
 - ³⁷ H. Saleur, in *Topology and Geometry in low dimensional physics*, edited by A. Comtet, T. Jolicœur, S. Ouvry, and F. David (Springer Berlin Heidelberg, 1998), vol. 69 of *Les Houches - Ecole d'Été de Physique Théorique*, pp. 473–550.

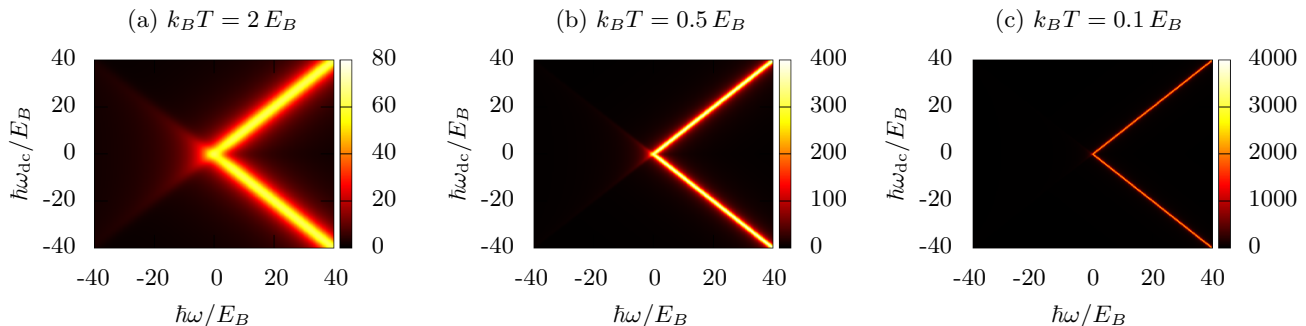


FIG. 5: (Color online) Infrared divergences for a good conductor at low temperature: Density plots of the ratio $\mathcal{S}_R(\omega, \omega_{\text{dc}})$ of the backscattering current noise for $R \neq 0$ to the same quantity at $R = 0$ (no environment) as function of $\hbar\omega/E_B$ and $\hbar\omega_{\text{dc}}/E_B$ for $R = 2R_q$ ($K = 1/3$) and various values of the temperature: (a) High temperature: $k_B T = 2 E_B$, (b) Lower temperature: $k_B T = 0.5 E_B$ and (c) Lowest temperatures: $k_B T = 0.1 E_B$. Frequencies ω and ω_{dc} are kept below the high energy cutoff $\hbar\omega_{\text{RC}} = 40 E_B$. Note the different color scales of each graph.

- ³⁸ C. de C. Chamon, D.E. Freed, and X.G. Wen, Phys. Rev. **B 51**, 2363 (1995).
- ³⁹ R. Sánchez, R. López, D. Sánchez, and M. Büttiker, Phys. Rev. Lett. **104**, 076801 (2010).
- ⁴⁰ U. Gavish, Y. Imry, Y. Levinson, and B. Yurke, in *Quantum noise in mesoscopic physics*, edited by Y. Nazarov (Kluwer Academic Publishers, 2003), vol. 97 of *NATO Science Series*, pp. 297–311.
- ⁴¹ R. Deblock, E. Onac, L. Gurevich, and L. Kouwenhoven, Science **301**, 203 (2003).
- ⁴² P.-M. Billangeon, F. Pierre, H. Bouchiat, and R. Deblock, Phys. Rev. Lett. **96**, 136804 (2006).
- ⁴³ J. Basset, H. Bouchiat, and R. Deblock, Phys. Rev. Lett. **105**, 166801 (2010).
- ⁴⁴ J. Basset, H. Bouchiat, and R. Deblock, Phys. Rev. B **85**, 085435 (2012).
- ⁴⁵ P. Fendley, A. Ludwig, and H. Saleur, Phys. Rev. **B 52**, 8934 (1995).
- ⁴⁶ P. Fendley, A. Ludwig, and H. Saleur, Phys. Rev. Lett. **75**, 4492 (1995).
- ⁴⁷ P. Fendley, A. Ludwig, and H. Saleur, Phys. Rev. Lett. **74**, 3005 (1995).
- ⁴⁸ H. Saleur and U. Weiss, Phys. Rev. **B 63**, 201302 (2001).
- ⁴⁹ A. Komnik and H. Saleur, Phys. Rev. Lett. **96**, 216406 (2006).
- ⁵⁰ L. Freton and E. Boulat, Phys. Rev. Lett. **112**, 216802 (2014).
- ⁵¹ P. Fendley, Adv. Theor. Math. Phys. **2**, 987 (1998).
- ⁵² P. Fendley and H. Saleur, Phys. Rev. Lett. **81**, 2518 (1998).