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Roberta Citro, Stefania de Palo, Nicolas Victorin, Anna Minguzzi, Edmond Orignac. Spectral Function of a Bosonic Ladder in an Artificial Gauge Field. *Condensed Matter*, 2020, 5 (1), pp.15. 10.3390/condmat5010015 . ensl-02503975

HAL Id: ensl-02503975

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Submitted on 6 Nov 2020

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Article

Spectral functions of a bosonic ladder in artificial gauge field

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Version November 6, 2020 submitted to *Condens. Matter*

Abstract: This is only the original submission, for the final version download the file from the MDPI Open Access Journal *Condensed Matter*. We calculate the spectral function of a boson ladder in an artificial magnetic field by means of analytic approaches based on bosonization and Bogoliubov theory. We discuss the evolution of the spectral function at increasing effective magnetic flux, from the Meissner to the Vortex phase, focussing on the effects of incommensurations in momentum space. At low flux, in the Meissner phase, the spectral function displays both a gapless branch and a gapped one, while at higher flux, in the Vortex phase, the spectral functions display two gapless branches and the spectral weight is shifted at a wavevector associated to the underlying vortex spatial structure. While the Bogoliubov theory, valid at weak interactions, predicts sharp delta-like features in the spectral function, at stronger interactions we find power-law broadening of the spectral functions due to quantum fluctuations as well as additional spectral weight at higher momenta due to backscattering and incommensuration effects. These features could be accessed in ultracold atom experiments using radio-frequency spectroscopy techniques.

Keywords: bosonization, Bogoliubov approximation, artificial gauge field, spectral functions

1. Introduction

In quasi one-dimensional systems, analogues of the Meissner and Vortex phase have been predicted for the bosonic two-leg ladder [1–4], the simplest system where orbital magnetic field effects are allowed. It was shown that in this model, the quantum phase transition between the Meissner and the Vortex phase is a commensurate-incommensurate transition [5–7]. Recently the advent of ultracold atomic gases, have opened a route where to realize low dimensional strongly interacting bosonic systems [8–10] where an artificial magnetic flux acting on the ladder can be simulated either using geometric phases [11], or the spin-orbit coupling [12,13]. Indeed, there is a mapping of the two-leg ladder bosonic model to a two-component spinor boson model in which the bosons in the upper leg become spin-up bosons and the bosons in the lower leg spin-down bosons. Under such mapping, the magnetic flux of the ladder becomes a spin-orbit coupling for the spinor bosons. Theoretical proposals to realize either artificial gauge fields and artificial spin orbit coupling have been put forward [14,15], and an artificial spin-orbit coupling has been achieved in a cold atoms experiment [16]. A two leg boson ladder under a flux is known to display a commensurate-incommensurate transition [1–4] between a low flux commensurate Meissner-like phase and a high flux incommensurate vortex-like phase. The transition has been characterized using equal time correlation functions [3,17–19]. However, we

31 expect a direct signature of the transition also in dynamical correlation functions. In one dimension,
 32 the low energy modes are collective excitations[20,21], and in the two-leg ladder, there is a separation
 33 between a total density (“charge”) and a density difference (“spin”) mode[2,4]. This is analogous to the
 34 well-known spin charge separation in electronic systems[20] and two-component boson systems[22].
 35 Except at commensurate filling[23–26] the “charge” mode is gapless. By contrast, the “spin” mode
 36 is gapped in the Meissner phase and gapless in the Vortex phase, the transition as a function of flux
 37 being in the commensurate-incommensurate class[5,6]. Thus, the two phases are characterized by
 38 very different dynamical correlation functions. Among those correlation functions, one could for
 39 example consider the “spin-spin” dynamical structure factor. This would display a well defined
 40 gapped or gapless dispersion respectively in the Meissner and in the Vortex phase. However, such
 41 correlation function would not be sensitive to the incommensuration in the weak interchain hopping
 42 regime, although it displays incommensuration features at weak interactions and large interchain
 43 hopping[4,27]. A better indicator of incommensuration in all regimes is provided by the spectral
 44 function of the bosonic particles. In the Vortex phase, it always displays a shift in the position of the
 45 minimum of the dispersion away from $q = 0$ as a consequence of the incommensuration whereas
 46 in the Meissner state the minimum of the dispersion remains at $q = 0$. A particular feature of the
 47 single-particle spectral function is that it is incoherent[22,28] i.e. the low energy excitation branches
 48 emerge as power law singularities instead of delta function singularities. From the experimental
 49 point of view, single-particle spectral functions are accessible via radiofrequency (RF) spectroscopy
 50 techniques [29,30]. In the present paper, we calculate the boson spectral function in the different phases
 51 of the boson ladder at incommensurate filling in order to fully characterize the transition under flux.

52 2. Model

53 In the following we use the notations and definitions of Ref. [19]. We consider a model of bosons
 54 on a two-leg ladder in the presence of an artificial U(1) gauge field[13,31]:

$$H = -t \sum_{j,\sigma} (b_{j,\sigma}^\dagger e^{i\lambda\sigma} b_{j+1,\sigma} + b_{j+1,\sigma}^\dagger e^{-i\lambda\sigma} b_{j,\sigma}) + \frac{U}{2} \sum_{j,\sigma} n_{j\sigma} (n_{j\sigma} - 1) + \frac{\Omega}{2} \sum_{j,\alpha,\beta} b_{j,\alpha}^\dagger (\sigma^x)_{\alpha\beta} b_{j,\beta}. \quad (1)$$

55 where $\sigma = \uparrow, \downarrow$ represents the leg index or the internal mode index[32–34], $b_{j,\sigma}$ annihilates a boson on
 56 leg σ on the j -th site, $n_{j\alpha} = b_{j\alpha}^\dagger b_{j\alpha}$, t is the hopping amplitude along the chain, Ω is the tunneling
 57 between the legs or laser induced tunneling between internal modes, λ is the flux of the effective
 58 magnetic field, U is the repulsion between bosons on the same leg. The low-energy effective theory
 59 for the Hamiltonian (1), where $\Omega \ll t$ is treated as a perturbation, is obtained by using Haldane’s
 60 bosonization.[35] By introducing[35] the fields $\phi_\alpha(x)$ and $\Pi_\alpha(x)$ satisfying canonical commutation
 61 relations $[\phi_\alpha(x), \Pi_\beta(y)] = i\delta(x-y)$ as well as the dual $\theta_\alpha(x) = \pi \int^x dy \Pi_\alpha(y)$ of $\phi_\alpha(x)$, and after
 62 introducing the respective combinations of operators $\phi_{c,s} = \phi_\uparrow \pm \phi_\downarrow$ we can represent the low-energy
 63 Hamiltonian as $H = H_c + H_s$, where

$$H_c = \int \frac{dx}{2\pi} \left[u_c K_c (\pi \Pi_c)^2 + \frac{u_c}{K_c} (\partial_x \phi_c)^2 \right] \quad (2)$$

64 describes the total density (or charge) fluctuations for incommensurate filling when umklapp terms
 65 are irrelevant, and

$$H_s = \int \frac{dx}{2\pi} \left[u_s K_s \left(\pi \Pi_s + \frac{\lambda}{a\sqrt{2}} \right)^2 + \frac{u_s}{K_s} (\partial_x \phi_s)^2 \right] - 2\Omega A_0^2 \int dx \cos \sqrt{2}\theta_s, \quad (3)$$

66 describes the antisymmetric density (or spin) fluctuations. In Eq. (2) and (3), u_s and u_c are respectively
 67 the velocity of antisymmetric and total density excitations, A_0 is a non universal coefficient[20] while
 68 K_s and K_c are the corresponding Tomonaga-Luttinger (TL) exponents[36]. For two chains of hard
 69 core bosons, we have $u_c = u_s = 2t \sin(\pi\rho^0/2)$ where ρ^0 is the average number of bosons per site and

70 $K_s = K_c = 1$.

71 The phase diagram of the Hamiltonian has been determined by looking at the behavior of the chiral
72 current, *i.e.* the difference between the currents in upper and lower leg, which is defined as

$$J_s(j, \lambda) = -it \sum_{\sigma} \sigma (b_{j,\sigma}^{\dagger} e^{i\lambda\sigma} b_{j+1,\sigma} - b_{j+1,\sigma}^{\dagger} e^{-i\lambda\sigma} b_{j,\sigma}), \quad (4)$$

$$= \frac{u_s K_s}{\pi\sqrt{2}} \left(\partial_x \theta_s + \frac{\lambda}{a\sqrt{2}} \right). \quad (5)$$

73 As a function of the flux λ , the chiral current first increases linearly with λ while being in the Meissner
74 phase and above a critical value of λ it starts to decrease in the Vortex phase[2]. In this phase the
75 rung current starts to be different from zero. In Fig.1 the red-line is the phase boundary between the
76 Vortex and the Meissner phase for the non-interacting case, while the blue-line represents the phase
77 boundary in the hard-core limit[19]. The major difference with respect to the non-interacting case is
78 the persistence of the Meissner phase even for large values of the flux.

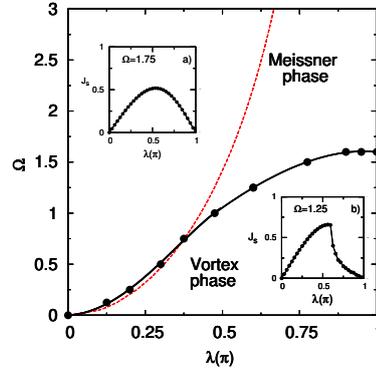


Figure 1. Phase diagram for a hard-core bosonic system on a ladder as a function of flux per plaquette λ and Ω , at the filling value $n = 1$. The black solid line that joins solid dots is the phase boundary between the Meissner and the Vortex phase, while the dashed red line is the prediction for this boundary in the non-interacting system. In the insets we show the different behavior of the spin-current $J_s(\lambda)$ for two values of interchain coupling Ω when there is the Meissner/Vortex transition and where there is not, respectively panel b) for $\Omega = 1.25$ and a) for $\Omega = 1.75$ DMRG simulation results at $L = 64$ in PBC.

79 Beyond the chiral and rung current the Meissner to Vortex phase transition can be traced out by
80 looking at the behavior of the spectral function which is more sensitive to incommensurations.

81

82 For the case of lattice bosons the spectral function is defined as:

$$A_{\sigma}(q, \omega) = -i \sum_j \int dt \theta(t - t') e^{i(qx_j - \omega t)} \left[\langle b_{j\sigma}(t) b_{0\sigma}^{\dagger}(0) \rangle - \langle b_{0\sigma}^{\dagger}(0) b_{j\sigma}(t) \rangle \right], \quad (6)$$

83 and can be experimentally accessed by, e.g. via radiofrequency (RF) spectroscopy techniques [29,30].
84 In the following we will focus on the positive-frequency part of the spectral function, given by the first
85 term in Eq. (6).

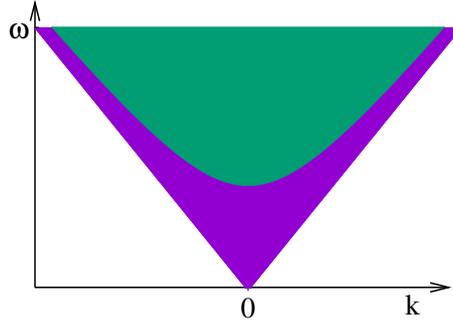


Figure 2. Schematic representation of the spectral function in the Meissner phase. The colored regions have a non-zero spectral weight. The violet region is the spectral weight coming only from the gapless charge modes, the spin modes remaining in their ground state. The green region represents the region where the gapped spin modes contribute to the spectral weight.

86 3. Spectral function in the Meissner phase for weak interchain hopping

Within the bosonization technique the boson annihilation operator to the lowest order approximation can be represented as:

$$\psi_\sigma(x, t) = b_{j,\sigma}(\tau) / \sqrt{a} \sim A_0 \langle e^{i\frac{\theta_s}{\sqrt{2}}} \rangle e^{i\frac{\theta_c(ja,\tau)}{\sqrt{2}}}, \quad (7)$$

where a is the lattice spacing, A_0 is a non-universal constant and σ stands for \uparrow in the upper chain and \downarrow for the lower chain. Knowing the Green's function for the single particle operators b one gets the spectral function as:

$$A(k, \omega) \sim A_0^2 |\langle e^{i\frac{\theta_s}{\sqrt{2}}} \rangle|^2 \int \frac{dxdt}{2\pi} e^{-i(kx-\omega t)} \left(\frac{\alpha^2}{(\alpha - iu_c t)^2 + x^2} \right)^{\frac{1}{8K_c}}, \quad (8)$$

where α is the theory cutoff taken equal to the lattice spacing. The result of the integral yields

$$A(k, \omega) \sim \frac{(A_0 \alpha)^2 \pi}{u_c} |\langle e^{i\frac{\theta_s(x,t)}{\sqrt{2}}} \rangle|^2 \frac{e^{2\frac{\omega}{u_c} \alpha}}{\Gamma^2\left(\frac{1}{8K_c}\right)} \left| \left(\frac{\omega^2}{u_c^2} - k^2 \right) \alpha^2 \right|^{\frac{1}{8K_c} - 1} \theta(\omega) \theta\left(\frac{|\omega|}{u_c} - |k|\right), \quad (9)$$

The approximation (11) only yields the behavior of the spectral function at ω lower than the gap Δ_s in the θ_s modes. The actual correlation function can be obtained from the Form factor expansion[37–41]. The lowest contribution, from a soliton-antisoliton pair yields

$$\langle T_\tau e^{i\frac{\theta_s(x,\tau)}{\sqrt{2}}} e^{-i\frac{\theta_s(0,0)}{\sqrt{2}}} \rangle = |\langle e^{i\frac{\theta_s(x,t)}{\sqrt{2}}} \rangle|^2 + O(e^{-2\Delta_s \sqrt{(x/u_s)^2 + \tau^2}}), \quad (10)$$

87 As a result, the Fourier transform of the full Matsubara Green's function is the sum of the
 88 contribution (9) and a second contribution analytic in a strip of the upper iv half plane of width
 89 proportional to the gap. This implies that the analytic continuation to real frequencies of this
 90 contribution is real until $\omega = 2\Delta_s$. There, a cut appears along the real frequency and the imaginary
 91 part of that contribution to the Green's function becomes nonzero. This behavior is represented
 92 schematically on Fig. 2. As the flux increases, the gap decreases linearly until it becomes zero at the
 93 commensurate-incommensurate point.

94 4. Spectral function in the Vortex phase for weak interchain hopping

In the vortex phase the boson field to the lowest order reads:

$$\psi_\sigma(x, t) = b_{j,\sigma}(\tau) / \sqrt{a} \sim A_0 e^{i\sigma q_0(\lambda)x_j} e^{i\frac{\theta_s(ja,\tau)}{\sqrt{2}}} e^{i\frac{\theta_c(ja,\tau)}{\sqrt{2}}}, \quad (11)$$

95 where $q_0(\lambda)$ is the incommensurate wavevector of the vortex phase.

96 Thus we find the Matsubara Green's function of the bosons in the form

$$\langle T_\tau b_{j,\sigma}(\tau) b_{0,\sigma}^\dagger(0) \rangle = A_0^2 e^{-i\sigma q_0(\lambda)ja} \left(\frac{a^2}{(ja)^2 + (u_c \tau)^2} \right)^{\frac{1}{8K_c}} \left(\frac{a^2}{(ja)^2 + (u_s \tau)^2} \right)^{\frac{1}{8K_s^*}} \quad (12)$$

and the spectral function is obtained by the integral:

$$A(k, \omega) \sim A_0^2 \int \frac{dx dt}{2\pi} e^{-i(kx - \omega t)} \left(\frac{\alpha}{\alpha - i(u_c t - x)} \right)^{\frac{1}{8K_c}} \left(\frac{\alpha}{\alpha - i(u_s t - x)} \right)^{\frac{1}{8K_s^*}} \left(\frac{\alpha}{\alpha - i(u_c t + x)} \right)^{\frac{1}{8K_c}} \left(\frac{\alpha}{\alpha - i(u_s t + x)} \right)^{\frac{1}{8K_s^*}}, \quad (13)$$

97 where $q_0(\lambda)$ is absorbed into k . The Fourier transform of the Matsubara Green's function (12) can be
98 calculated by the method outlined in [22,28] and after analytic continuation $iv \rightarrow \omega + i0_+$ it reads:

$$G_\sigma(q, \omega) = \left(\frac{a}{2} \right)^{\frac{1}{4K_c} + \frac{1}{4K_s^*}} \frac{\Gamma\left(1 - \frac{1}{8K_c} - \frac{1}{8K_s^*}\right)}{\Gamma\left(\frac{1}{8K_c} + \frac{1}{8K_s^*}\right)} |\omega^2 - u_s(q + \sigma q_0(\lambda))|^{\frac{1}{8K_c} + \frac{1}{8K_s^*} - 1} e^{i\pi\left(1 - \frac{1}{8K_c} - \frac{1}{8K_s^*}\right)} \Theta(\omega^2 - u_s(q + \sigma q_0(\lambda))^2) \\ \times \frac{1}{u_s^{\frac{1}{4K_c} + \frac{1}{4K_s^*} - 1}} F_1\left(\frac{1}{8K_c}, \frac{1}{8K_c} + \frac{1}{8K_s^*} - \frac{1}{2}, 1 - \frac{1}{8K_c} - \frac{1}{8K_s^*}, \frac{1}{8K_c} + \frac{1}{8K_s^*}; 1 - \frac{u_c^2}{u_s^2}, 1 - \frac{\omega^2 - u_c^2(q + \sigma q_0(\lambda))^2}{\omega^2 - u_s^2(q + \sigma q_0(\lambda))^2}\right), \quad (14)$$

99 where the function $F_1(a, b_1, b_2, c; z_1, z_2)$ is an Appell hypergeometric function[42], which has a series
100 representation in terms of two complex variables z_1 and z_2 when $|z_1| < 1$ and $|z_2| < 1$.

101 Singularities appear at $\omega_1^\sigma(q, \lambda) = \pm u_c(q + \sigma q_0(\lambda))$ and at $\omega_2^\sigma(q, \lambda) = \pm u_s(q + \sigma q_0(\lambda))$ and the
102 power-law behavior of the spectral function near these points has been detailed in Ref. [28]. Some
103 attention should be paid to extract the analytic continuation for points outside the radius of convergence
104 of the Appell's function resorting to its integral representation possible when $\Re[c - a] > 0$ which in
105 our case is always true by construction: $c - a = 1/(8K_s^*)$. The behavior of the Green's function near the
106 singularity points can be simplified as:

$$G_\sigma(q, \omega) \simeq |\omega^2 - \omega_1^\sigma(q, \lambda)|^{1/(8K_c) + 1/(4K_s^*) - 1} \quad (15)$$

$$G_\sigma(q, \omega) \simeq |\omega^2 - \omega_2^\sigma(q, \lambda)|^{1/(8K_s^*) + 1/(4K_c) - 1} \quad (16)$$

107 In the Vortex phase, near the commensurate-incommensurate transition the spin velocity $u_s^* \propto \sqrt{\lambda - \lambda_c}$,
108 so we stay with the case where the charge velocity is larger than the spin one: in this case $1 - u_c^2/u_s^2 \leq 0$
109 and $\omega_2^\sigma(q, \lambda) \leq \omega_1^\sigma(q, \lambda)$. In this phase $K_s^* > 1/2$ and K_c is near unity, when the hopping between
110 the chains is not too large, the imaginary part of the Green's function, *i.e.* the spectral function
111 $A_\sigma(q, \omega) = -\Im m G_\sigma(q, \omega) / \pi$, is divergent near the two poles ω_1 and ω_2 as shown in panel *a*) of Fig. 3.
112 In order to wash out at least one of the divergencies near the two poles small values of $K_c < 1/2$ are
113 required, signalling that density wave correlations are becoming important and eventually bringing a
114 density-wave phase. The behavior of the spin resolved spectral function $A_\sigma(q, \omega)$ for a fixed value of
115 the applied flux is schematically shown in Fig. 4 as a function of the q and ω showing the contribution
116 to spectral weight coming from the different singularities.

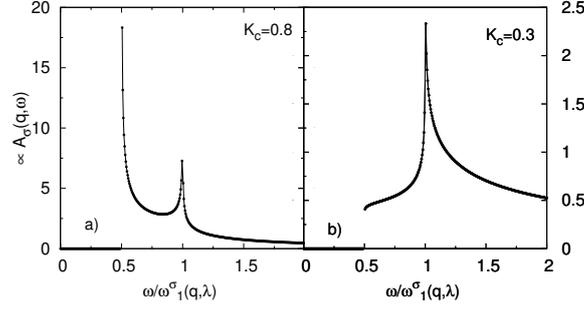


Figure 3. Spectral function $A_\sigma(q, \omega)$ as a function of $\omega/\omega_1^\sigma(q, \lambda)$ for $u_s^*/u_c = 0.5$ and $K_s^* = 0.6$. In panel *a*) we show the typical situation in the Vortex phase ($K_c = 0.8$) while in panel *b*) we show the case $K_c = 0.3$

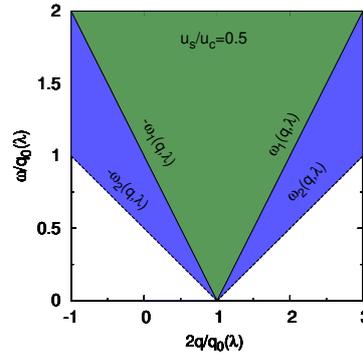


Figure 4. Schematic representation of the spectral function $A_\sigma(q, \omega)$ as a function of ω and q for a fixed applied field λ inducing a finite $q_0(\lambda)$ for $u_s^*/u_c = 0.5$. Finite spectral weights are present only in the colored region. In the blue region there is only the contribution from the singularity at $\omega_2(q, \lambda)$, while in the green one the contribution from $\omega_1(q, \lambda)$ adds up.

117 5. Spectral functions in the weakly interacting regime from bosonization

118 We adopt here an alternative bosonization scheme[4], valid at weak interactions but arbitrary
 119 inter-leg tunnel coupling Ω . In this regime, one can bosonize starting from the exact single-particle
 120 excitation spectrum [4] which displays a single minimum in the Meissner phase and two minima in the
 121 Vortex phase. In the Meissner state, the result (9) is recovered, but by construction of the bosonization
 122 scheme, the contribution of gapped modes at higher energy is not accessible.

123 In the Vortex phase, at low energy the field operators are approximated as [4]

$$\begin{aligned} b_{j\uparrow} &= u_Q \beta_{j+} e^{-iQj} + v_Q \beta_{j-} e^{iQj} \\ b_{j\downarrow} &= v_Q \beta_{j+} e^{-iQj} + u_Q \beta_{j-} e^{iQj} \end{aligned} \quad (17)$$

124 where u_Q and v_Q are the single-particle amplitudes which diagonalize the non-interacting ladder
 125 Hamiltonian, calculated at the minima $\pm Q$ of the lower branch dispersion relation, and $\beta_{j\pm} =$
 126 $\sum_q e^{-iqja} \beta_{q\pm Q}$ with β_k being the destruction operator of the lower single-particle excitation branch.

127 Then, the field operators are bosonized as $\beta_{j\pm} = \sqrt{\bar{n}}e^{i\theta_{\pm}(x_j)}$ and the Luttinger liquid Hamiltonian
 128 takes the usual quadratic form in the symmetric, antisymmetric sectors corresponding to the operators
 129 $\theta_{s(a)} = (\theta_+ \pm \theta_-) / \sqrt{2}$. The associated Luttinger parameters are called K_s, v_s, K_a, v_a .

The Green's function, calculated *e. g.* for the upper leg $\sigma = 1/2$ reads

$$G_{\uparrow}(j, 0) = \langle b_{j\uparrow}(t)b_{0\uparrow}^{\dagger}(0) \rangle = u_Q^2 \langle \beta_{j+}(t)\beta_{0+}^{\dagger}(0) \rangle e^{-iQj} + v_Q^2 \langle \beta_{j-}\beta_{0-}^{\dagger}(0) \rangle e^{iQj} \quad (18)$$

From bosonization we obtain

$$\langle \beta_{j\pm}(t)\beta_{0\pm}^{\dagger}(0) \rangle = \bar{n} \left(\frac{a^2}{(ja)^2 - (v_s t)^2} \right)^{1/(8K_s)} \left(\frac{a^2}{(ja)^2 - (v_a t)^2} \right)^{1/(8K_a)} \quad (19)$$

130 while $\langle \beta_{j\pm}(t)\beta_{0\mp}^{\dagger}(0) \rangle = 0$. This can be Fourier transformed as done in Sec.4, yielding a spectral
 131 function with two incoherent contributions at $q = \pm Q$, each with a power law singularity at the two
 132 excitation branches $\omega = v_{s,a}|q \pm Q|$. The same result is obtained in the lower leg, up to an exchange of
 133 u_Q^2 and v_Q^2 .

134 6. Spectral function in the Bogoliubov theory

135 In the previous sections we have derived the expressions for the spectral function with the
 136 bosonization technique, valid at intermediate and strong interactions. In the regime of very weak
 137 interactions and large filling of the lattice, a complementary approach is provided by the Bogoliubov
 138 theory [27]. The system is described by a two-component Bose-Einstein condensate with wavefunction
 139 $\Psi_{j\sigma}^{(0)}$ and small fluctuations on top of it. The condensate wavefunction $\Psi_{j\sigma}^{(0)}$ is obtained by solving the
 140 coupled discrete non-linear Schroedinger equations

$$\begin{aligned} \mu \Psi_{l,1}^{(0)} &= -t \Psi_{l+1,1}^{(0)} e^{i\lambda} - t \Psi_{l-1,1}^{(0)} e^{-i\lambda} \\ &\quad - (\Omega/2) \Psi_{l,2}^{(0)} + U |\Psi_{l,1}^{(0)}|^2 \Psi_{l,1}^{(0)} \\ \mu \Psi_{l,2}^{(0)} &= -t \Psi_{l+1,2}^{(0)} e^{-i\lambda} - t \Psi_{l-1,2}^{(0)} e^{i\lambda} \\ &\quad - (\Omega/2) \Psi_{l,1}^{(0)} + U |\Psi_{l,2}^{(0)}|^2 \Psi_{l,2}^{(0)}, \end{aligned} \quad (20)$$

where μ is the chemical potential. The field operator is approximated by

$$b_{j\sigma}(t) \simeq \Psi_{j\sigma}^{(0)} + \sum_{\nu} h_{\nu j}^{\sigma} \gamma_{\nu} - Q_{\nu j}^{\sigma*} \gamma_{\nu}^{\dagger}, \quad (21)$$

141 where $h_{\nu j}^{\sigma}$ and $Q_{\nu j}^{\sigma}$ are the Bogoliubov mode wavefunctions with energy ω_{ν} and γ_{ν} are the quasiparticle
 142 creation and destruction field operators, satisfying bosonic commutation relations (see [27] for the full
 143 expressions).

Using the definition (6) for the spectral function together with the mode expansion of the bosonic
 field operators (21) we obtain

$$A(q, \omega) = - \sum_{\nu} [|\tilde{h}_{\nu q}^{\sigma}|^2 \delta(\omega - \omega_{\nu}) - |\tilde{Q}_{\nu q}^{\sigma}|^2 \delta(\omega + \omega_{\nu})] \quad (22)$$

144 where $\tilde{h}_{\nu q}^{\sigma} = \sum_j e^{-ikaj} h_{\nu j}^{\sigma}$ and $\tilde{Q}_{\nu q}^{\sigma} = \sum_j e^{-ikaj} Q_{\nu j}^{\sigma}$.

145 The spectral function in the Bogoliubov approximation is illustrated in Figure...

146 7. Conclusion

147 We have obtained the spectral functions of a two-leg boson ladder in an artificial gauge field. The
 148 bosonization approach, describing the regime of sufficiently strong interactions, predicts that in the
 149 Meissner phase, the low energy spectral weight is located near $\omega = 0, q = 0$. In the Vortex phase,

150 it is located near $\omega = 0, \pm q_0(\lambda)$. In both cases, the spectral weight is incoherent and characterized
 151 by power law singularities at $\omega = u_c|q|$ (Meissner phase) or $\omega = u_c|q \pm q_0(\lambda)|$ (Vortex phase) with
 152 known exponents, and a specific incommensuration effect due to the shift of the spectral weight for
 153 $q \simeq q_0(\lambda)$. In the Meissner phase, the gap in the antisymmetric density fluctuations translates as a
 154 power law singularity of the spectral function at frequency $\omega > 2\Delta_s$. The Bogoliubov approximation,
 155 valid at weak interactions predicts delta-like spectral function, still keeping the main features: a
 156 single gapless excitation branch and a gapped one in the Meissner phase and two gapless excitation
 157 branches displaying incommensuration effects in the Vortex phase. The Bogoliubov theory misses the
 158 backscattering contributions to the spectral function, consistently with the bosonization predictions
 159 that their spectral weight is very small at weak interactions.

160 The present work could be extended in different directions. Exactly at the
 161 commensurate-incommensurate transition, the antisymmetric excitations are described by a
 162 gapless theory[43] with dynamical exponent $z = 2$. The finite temperature correlation function has a
 163 known scaling form[44,45], and the spectral function at the commensurate-incommensurate transition
 164 can be obtained by convolution of that correlation function with the one of the charge modes. Such
 165 calculation is left for future work. Another possible extension is to consider the interleg interaction.
 166 Previous work has shown[36,46] that it splits the commensurate incommensurate transition point
 167 into an Ising transition point, a disorder point and a Berezinskii-Kosterlitz-Thouless (BKT) transition
 168 point. An intermediate atomic density wave exists between the Ising and the BKT point, and it
 169 develops incommensuration at the disorder point. The atomic density wave could be characterized
 170 using the spectral functions as done in the present manuscript, both in its commensurate and in its
 171 incommensurate regime. A final possible extension is to consider the spectral functions in the presence
 172 of the second incommensuration[19,47] at $\lambda = \pi n$.

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